THE PARAMETERIZATION OF DEEP CONVECTION

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Abstract Insights into the parameterization of convection from two decades of diagnostic studies are reviewed. The life cycle of a convective mesosystem mass flux is described using day 245 from the GARP Atlantic Tropical Experiment as example. The thermodynamic differences between non-precipitating convection and precipitating convection are discussed, as well as the importance of the mid-tropospheric freezing level as the tropics and the saturation pressure budget in the tropics. The strengths and weaknesses of the mass flux parameterization of deep convection is discussed. Diagnostic studies, which have identified three key vertical modes in the convective heating and drying structure, are outlined. Two are related deep modes associated with precipitation and deep tropospheric ascent, but a variable upward equivalent potential temperature (θ_e) flux. The other is a double mode structure with ascent in the upper troposphere over descent in the lower troposphere, coupled to inflow at the freezing level, with no net precipitation or transport of θ_e the mode associated with deep mesoscale arval. We discuss the mass flux formulation of convective updrafts and downdrafts. We outline the concepts (but not the details) behind the Bates-Miller parameterization and suggest two extensions. One is a formulation of the adjustment time in terms of grid-scale and gravity-wave propagation speed for the two primary modes. The second is an explicit parameterization of the mesoscale arval mode.

1. Introduction

There have been several reviews of convective parameterization in large-scale models including Betts [1], Frank [2], text-books such as Cotton and Anthes [3] and Emanuel [4], and particularly the recent American Meteorological Society monograph by Emanuel and Raymond [5]. This includes a discussion of the Betts-Miller scheme [6], which includes a formulation for unstratified downdrafts driven by convection. Although I shall summarize the basis of the Betts-Miller scheme in section 4, and suggest a further extension, I will not discuss the details. For the most part, I shall review here some of the fundamental issues that have been uncovered by many researchers over the last twenty-five years and try to reduce them to their simplest form. As a result this paper is part history, part review and part reinterpretation, together with a few suggestions for a way forward. This paper discusses only the energy and water transports by convection, not the issues of the momentum and

vorticity transports 1 only address the closure problem in the limited case of the Betts-Miller scheme in section 4

2. Deep Cumulus Diagnostic results

2.1 DEVELOPMENT OF CUMULUS PARAMETERIZATIONS

The development of cumulus parameterizations, which started in the 1960s with the work of Kra [7] and Manabe and Stieglitz [8] and two important conceptual papers by Ooyama [9] [10], received a stimulus from diagnostic models in the 1970s. The Arakawa and Schubert [11] paper and the diagnostic study of Yamas [12], using data over the tropical Pacific, were closely paralleled. Their cloud model based on entraining updrafts has dominated this school of cumulus parameterization up to this date. They showed how the flux could be calculated, independent of the precipitation, using this cloud mass flux model. Subsequently this model has been extended to include convective downdrafts Arakawa and Cheng [13]. At the same time as [11][12], there were two parallel papers by Betts [14] [15]. The first [14] introduced the mixed layer model for the dry boundary layer and the subcloud layer, which was included in the Arakawa-Schubert parameterization. This paper also extended the use of conserved variables (in particular, the liquid water potential temperature, dealing with the coupling of the enthalpy and liquid water fluxes in convection), and discussed the concept of lapse rate equilibrium for shallow convection. The second paper [15] used data from the first Venezuela International Microphysical and Hydrological Experiment (VIMEX) to formulate an updraft-downdraft budget model for mesoscale cumulonimbus systems, based on composite raingauge observations in relation to radar echoes. It showed how using flux conservation separately for the updraft and downdraft circulations permitted a separate evaluation of updraft condensation, evaporation into the downdraft, and net precipitation. Figure 1a shows the mass fluxes associated with the high $\theta_v$ updraft circulation and the low $\theta_v$ downdraft circulation. The paper concluded that half the condensation in the updraft was evaporated into the downdraft. The composite mesoscale system was also partitioned temporally into the growth and decay phase (Figure 1b). This showed how the net lifecycle upward mass flux was the result of an upward mass circulation in the growth phase, and a downward circulation in the decay phase. The paper also showed that the low level transformation by deep convection was dominated by cold dry downdraft outflows. The data used by [15] was quite primitive, but the subsequent tropical field programs have supported the conclusions of the analysis. The second VIMEX experiment explored in more detail the transports by unstratified downdrafts [16] and these modeling (Betts and Silva Dias [17]) Miller and Betts [18] used a mesoscale model to study the distinction between convective and mesoscale downdrafts, reaching similar conclusions to pioneering papers by Zipser [19] [20] and the GATE squall line study of Houze [21].

2.2. GATE DIAGNOSTIC STUDIES

By the end of that decade, it was clear from these tropical studies over land and from the Global Atmospheric Research Program (GARP) Atlantic Tropical Experiment (GATE) (see reviews by Betts [22], Houze and Betts [23]) that parameterizing tropical convection needed a model for the lifecycle effect of mesoscale convection, which evolves from lines of cumulonimbus to mesoscale convective systems within 6-12 hours, on horizontal scales that are not well resolved in global models.

Figure 2. Schematic of GATE convective band development.

Figure 2 is a schematic, typical of the GATE environment with strong shear in the low levels. Within the time-span of a typical mesoscale aircraft mission, the observer would see the development of cumulonimbus bands oriented along the low level shear, with inflow on one side and a developing anvil outflow to the rear. A typical spacing might be 60 km as each line evolved, the strong convection would decay, and a deep mesoscale anvil would persist often for many hours, with steady stratiform rain and a characteristic bright band at
the freezing level (Leary and Haute [24] [25] [26]). On some occasions, faster moving cross-shore lines with trailing arsks, and some of the characteristics of squall-lines, were observed (Haines and Stockman, [27]). It was also clear from the detailed analyses of Ooyama in the late 1970s, that the freezing level, which in the tropics is at the mid-troposphere, is dynamically significant. In the mesoscale arsuk stage, it is the level of maximum convergence, with ascent above in the region of condensation and freezing, and descent below driven by the evaporation and melting of falling precipitation. However, although the analysis theory was published later in Ooyama [28], only some of the fascinating lifecycle case studies have been fully published. (The Ooyama, Chu and Ebenstein wind and thermodynamic analyses are available in an archive at the National Center for Atmospheric Research: documentation is available in Ebenstein and Ooyama [29]; Tollerud and Ebenstein [30]; and Ebenstein et al. [31] published some composites of mass, heat and moisture budgets of non-squall clusters based on that data. Other GATE composites were published (Frank [32]; Johnson [33]), based on larger scale analyses, but they do not show the mesoscale lifecycles well, because of the smoothing involved in the compositing. The interpretation of diagnostic budgets by Johnson [34] in terms of convective and mesoscale components was however a significant step forward. The most detailed analysis of the Ooyama, Chu and Ebenstein dataset was published by Cheng [35] [36] and Cheng and Yama [37]. Using an Anekaeva-Schubert type cloud model with a spectrum of entraining updrafts and downdrafts, together with a mesoscale circulation, Cheng [35] first explored a tilted updraft model, and then systematically determined mass flux distributions for convective-scale updrafts and downdrafts. Cheng and Yama [37] used the results from the cloud model to extract a mesoscale mass flux and heating field, which explicitly show the mesoscale couplet of heating and drying over cooling and moistening for several GATE cloud clusters. I will return to their analysis in section 3.7.

2.3 GATE CLOUD CLUSTER LIFECYCLE ON DAY 245

As a brief review, I will present an example here for the life cycle mass flux evolution of a major GATE cloud cluster on Julian Day 245. In the published literature, Bem [22] gave a preliminary analyses of the mass flux evolution for this day, Mower et al. [38] analysed the synoptic, radar and aircraft data for this day, and Wasaer [39] analyzed the cloud fields. This cluster evolution is an excellent example, as it occurred over the GATE ship array, and the evolution can be seen in both time and space. Bands of convection developed during the day, evolved into a mesoscale complex and then decayed. The daytime evolution over the ship array was studied by a stack of five aircraft [38], for which the present author was the airborne mission scientist. This cluster is case 31 in [37].

Figure 3 shows a time-series (from the Ooyama analysis) of the mean vertical motion, \( \omega \), from 0300 UTC to 2400 UTC on September 2, 1994 (Julian Day 245) at 3 hour intervals at 8.5°N, 22°W, just east of the center of the GATE ship array. The mass field showed upward vertical motion initially peaking in the lower troposphere at 0300 UTC. By noon, there was a strong ascent through the whole troposphere with the peak vertical motion in the middle troposphere. Then the upward motion strengthened further in the troposphere (1500 and 1800 UTC), as it decayed in the lower troposphere. This is the stage when strong mesoscale ascent develops in the extensive arsks in the upper troposphere, and induces a mesoscale downdraft forms, driven by the melting and evaporation of falling precipitation.

![Time-series of the mean vertical motion at 8.5°N, 22°W from 0300 to 2400 UTC on September 2, 1994 (Day 245), from Ooyama's analysis of GATE wind sounding data](image)

Figure 3: Time-series of the mean vertical motion at 8.5°N, 22°W from 0300 to 2400 UTC on September 2, 1994 (Day 245), from Ooyama's analysis of GATE wind sounding data.

![Horizontal section of wind field and divergence at 500 mb near the freezing level at 2100 UTC on day 245, showing peak convergence of 2.0 \( 10^4 \) s\(^{-1}\). The dotted lines are the GATE ship array and two A/B scale ship arrays. Arrows are indefinite and logarithmic. Data is from Ooyama's analysis of GATE wind sounding data](image)

Figure 4: Horizontal section of wind field and divergence at 500 mb near the freezing level at 2100 UTC on day 245, showing peak convergence of 2.0 \( 10^4 \) s\(^{-1}\). The dotted lines are the GATE ship array and two A/B scale ship arrays. Arrows are indefinite and logarithmic. Data is from Ooyama's analysis of GATE wind sounding data.
layer equilibrium has closely a mixing line structure, and liquid water is carried with air parcels, the convective transports can be well represented by a single mass flux (Sobel and Capper [44]). However, once cloud droplets grow large enough to fall out of air parcels into unsaturated air, the cloud microphysics becomes important, and the entire thermodynamic picture becomes more complex.

3.2 PRECIPITATING CONVECTION AND THE LIMITATION OF MASS FLUX MODELS

Firstly, determining the fraction of the precipitation that falls out as important. \( \Phi_0 \) is not longer conserved in the updrafts, and the subsequent thermodynamics of updraft parcels is different, since their cloud water is reduced considerably. Betts [45] proposed a simple precursor-scale parameterization for precipitation fallout, which gives asymptotic values for updraft cloud water. Even in small trade-cumulus clouds, once they become deep enough to precipitate, the fallout of precipitation produces layers in the atmospheric structure. The reason is that downdrafts formed by the evaporation of rain penetrate vertically more than the updrafts, which are terminated by condensation. The mixed esctems form by mixing, simply because, for the same change of mixing ratio, q, virtual potential temperature, \( \Theta \), changes faster along the moist adiabat (which is conserved in the evaporation of falling rain). Thus the warm adiabatic mixing, which allows for the transfer of moist energy, is more effective. By definition, mixing droplets are larger, they have a smaller surface to mass ratio, and they do not evaporate fast enough to keep the air into which they fall saturated, since this air, once cooled by evaporation, starts in downdrafts, seeking a new level of buoyancy equilibrium. In this process, downdraft subar is a result of an internal balance, which can be formulated conceptually in terms of a pressure scale for evaporation (Betts and Sobel [47], Betts [45]).

\[
\Pi' = \rho' \omega' \tau \quad (1)
\]

where \( \Pi' \) is a characteristic downdraft speed, and \( \tau \) is a characteristic evaporation time-scale for the water flux of the falling droplets, dependent on their microphysical properties, mean size and number density. The significance of \( \Pi' \) is that downdraft subaration, formulated in terms of the saturation pressure difference, \( \Pi' / \rho' \), is an air parcel saturation pressure \( \Pi' \) [41], asymptotically approaches \( \Pi' \) (see [17] and [43]). For the simple case of a uniform population of N drops of size \( r \), one can show (Kamburova and Liofdin [47], [17])

\[
\tau' \approx 4 \Pi' \Omega' N_r \quad (2)
\]

where \( \Omega' \) is the coefficient of diffusion of water vapor in air, and \( C_r \) is a ventilation coefficient for the evaporation of falling drops. The rainfall rate, downdraft speed and negative buoyancy and the stratification are all interactive, but typically we observe in cumulus/mixed downdraft outbreaks, subarations which correspond to values of \( \Pi' \) from 35-120 mb, corresponding to low level relative humidities from 85-95%. In unstable atmospheres
where down draught speeds are large, we tend to see higher values of $\dot{\Phi}$, corresponding to lower relative humidity.

The key consequence of (1) is that, unlike most updraughts, which remain close to saturation, the subsaturation of downdraughts depends on small-scale dynamical and microphysical parameters, which must be formulated in terms of large-scale model variables. This is in sharp contrast to the updraught circulation, which is very close to saturation (because the droplets are small and supersaturations are small, see for example Ludlam, [48]). Diagnostically if we measure downdraught inflow and outflow we can infer evaporation into them (e.g. Betts [15] [16]), but only a few parameter schemes (e.g. Emanuel [49], Betts and Miller [6]) have attempted to treat unsaturated downdraughts, even in a simple manner. Unfortunately in many convective parameterizations, most downdraughts, if included at all, are formulated simply as a fraction of the updraught mass flux, and often treated as saturated. This successfully avoids the real complexity of the deep convective process by reducing the parameter problem to determining a single updraught mass flux, but it is an unsatisfactory simplification. Unlike shallow convection, deep convection is not a simple mass flux problem, because the precipitation is falling freely and interacting with the atmosphere. For both the updraught and downdraught circulations, only one variable $\eta$ (approximately conserved), and the submodels which handle the microphysics of precipitation fallout and evaporation into downdraughts are critical. The widespread use of mass flux models for cumulus parameterization has perhaps partially obscured this important issue. For precipitating convection, the true thermodynamics of both updraught and downdraught are poorly determined, because of this dependence on the microphysics. The updraught may be saturated (which gives the illusion of simplicity), but its cloud water (which determines its $\beta$) is dependent on the microphysics. This is the reason Betts [45] proposed a symmetric most useful budget for the study of precipitating convection, after the fallout of precipitation. Parallel to (1), one can relate a pressure scale for precipitation fallout, $\dot{\Pi}$, to an updraught speed ($W_U$) and a timescale for the conversion of cloud water or ice to precipitation ($\tau_p$).

$$\dot{\Pi}_U = g\rho_0 \gamma_p \tau_p$$

The conversion timescale, $\tau_p$, is of course not easy to estimate, as the underlying microphysics is complex, but the formulation is useful conceptually, because the saturation pressure difference $p_s - p_c$ (for a steady-state updraught) approximately approaches $\dot{\Pi}_U$. Consequently, cloud water, $\dot{\Pi}_U$, is also asymptomatically related to $\dot{\Pi}$, Betts [45]

$$\dot{\Pi}_U = g\rho_0 \gamma_p \beta \dot{\Phi}_U$$

For $\dot{\Pi}_U = 50$ mb, $t = 1$ day, $g = 10$ m/s, and $\dot{\Phi}_U = 0.05$, these two pressure scales ($\dot{\Pi}_U$ and $\dot{\Phi}_U$) symbolically represent the key microphysical processes in convective updraughts and downdraughts, which essentially determine the saturation pressure of updraught and downdraught outflows.

1.3 The Importance of the Saturation Pressure Budget

The saturation pressure ($p_s$) budget (see [41] [50] [51] [52]) is probably conceptually the second most important budget in the study of precipitating convection, after the $\dot{\Phi}_U$ budget. It can be regarded as another combination of the temperature and humidity budgets with important conservation properties for updraughts, saturation pressure difference, $p_s$, is related to cloud water content. Expression (4) is a special case of

$$\dot{\Pi}_U = g\rho_0 \gamma_p \beta \dot{\Phi}_U$$

(5a)

For downdraughts and the unsaturated environment, $\dot{\Pi}_U$ is related to relative humidity ($\dot{\Phi}_U$). For unsaturated air, a parallel relationship to (5a) exists:

$$\dot{\Pi}_U = g\rho_0 \gamma_p \beta \dot{\Phi}_U$$

(5b)

where $\gamma_p$ is saturation mixing ratio at the wet-bulb temperature [41], but a useful simple formulation of the direct relationship to $\dot{\Phi}_U$ is (see Appendix):

$$\dot{\Pi} = \rho \dot{\Phi} \rho \dot{\Phi} - (\dot{\Phi} - \dot{\Phi}_U)$$

(6)

where the thermodynamic coefficient

$$A = \frac{\rho}{\rho_0} = \frac{\rho}{\rho_0} \left( \frac{T}{2C\rho} \right)$$

(7)

increases slowly with decreasing temperature, increasing the ratio of the gas constants ($\dot{\Phi}/\dot{\Phi}_U$), as well as the latent heat ($\dot{\Phi}/\dot{\Phi}_U$) and specific heat of air ($\dot{\Phi}/\dot{\Phi}_U$) given. At $T = 273$ K, $\rho_0$ is close related measures of saturation.

The vertical transport of water in the atmosphere is accomplished by clouds, and the outflows from convective systems. The transition at cloud boundaries, where the air is just saturated, and $\rho_\phi = \rho$, is in addition critical from the radiative perspective. The residual cloud water or ice in updraught outflows tends to bring the atmosphere closer to saturation (since $\rho_\phi > \rho$) in the upper troposphere, where moisture tends to sink less in the more stable stratification at these levels. The evaporation of falling precipitation produces unsaturated downdraught outflows in the lower troposphere. If pressure scales are used to parameterize the cloud microphysical processes controlling precipitation and evaporation in downdraughts, then these convective/microphysical pressure scales, together with the convective mass outflows (and large-scale advection), control the local saturation pressure budget of the atmosphere. Outside cloud systems, adiabatic processes conserve saturation pressure, $p_s$, so that only the diabatic radiative process modifies $p_s$. [46]. The equations in [46] contain errors, so a summary is given here. As air rises with radiative cooling, we may write the rate of change of $p_s$ along trajectories as

$$\dot{p}_s = -\omega$$

(8)

The right hand terms can both be related to the radiative cooling rate, $\dot{\omega}$. Saturation pressure

$$\dot{p}_s = \dot{\omega}$$

(9)

where the thermodynamic gradient is along lines of constant saturation mixing ratio $q$. For air parcel $\omega$, a radiatively balanced subsidence rate is...
where the gradient is set as the mean stratification. Typical values in the tropics just above the tradewind layer (600–800 mb) are $\omega \approx 10^{-2}$ and $\omega = 10^{-3} \text{ d}^{-1}$. So, so that

$$\omega > \mu'$$

(11)

and as air parcels outside clouds sink in the free troposphere with radiative cooling, they become more unstable, because $\mu' = \mu' - \omega < 0$. Above 700 mb, with $\omega \sim 1 \text{ d}^{-1}$, stratospheric increases at a rate $\sim 20 \text{ mb day}^{-1}$. The subsidence of air in the subsiding branch of atmospheric circulation, which is an important radiative control on the climate in the tropics [41, 55], is thus linked to the mass circulation, and the time air has subsided with radiative cooling after crossing a cloud boundary.

3.4 THE FREEZING LEVEL

The freezing level is important in the tropics; it is in the middle troposphere near 550 mb. Usually it is the level where the thermal profile changes from unstable to saturated equivalent potential temperature, $\theta_{e}$, to stable [41]. Often there is a visible kink in the thermal $\theta_{e}$ structure, and typically the level of minimum $\theta_{e}$ is near the freezing level. This has been known for many years, but it was not explicitly incorporated as a feature in a convective parametrization scheme until Betts [54], Betts and Miller [55]. Undoubtedly the strait precipitation phase change plays a role in the maintenance of the characteristic structure. We have already mentioned in section 2, that in the decay phase of convective mesosystems, the inflow peaks at the freezing level (Figure 4) with ascending motion above, and descending motion below. We shall discuss this convective mode further in section 3.5, and the relationship of the mid-tropospheric $\theta_{e}$ minimum to the upward $\omega$ flux in section 3.6.

3.5 THE KEY CONVECTIVE MODES

An important paper by Johnson [34] showed one method of partitioning diagnostic heat and moisture budgets into convective and mesoscale components. He assumed that the condensation in the mesoscale arvols was a certain fraction ($<0.2$) of the total precipitation, and then derived the characteristic mesoscale cloud signature of the warming and drying above the freezing level and cooling and moisturizing below.

This paper was followed by a review by Arakawa and Chen [56] which contained some significant diagnostic insights. They distinguished different types of closure assumptions in parameterization schemes; its parameter, they classified a Type II closer as one that constrained the coupling of the convective heat source ($Q_{c}$) and moisture sink ($Q_{m}$), using the notation of Yanai et al. [57], in which $Q_{c}$ is the total diabatic source term, and $Q_{m}$ is the radiative contribution to this term. They used canonical correlation analysis on the GATE Phase III data of Ooyama, Chu and Friborg, and an Asian data set from He et al. [57] to show there were three principal modes of coupling of ($Q_{c}$) and ($Q_{m}$). We show these schematically in Figure 5a, and 5b. Mode 1 is the principal deep convection mode associated with cumulonimbus updrafts and downbursts through the deep troposphere. (We will associate their third mode with a modulation of Mode 1: see below). Since there is

heating throughout the atmosphere and net precipitation, this mode is associated with a single-cell mode of mean upward vertical motion in the troposphere, although within that there are

moist updrafts and downdrafts. There is a net upward flux of $\theta_{e}$, peaking in the mid-troposphere, where the ($Q_{c}$) and ($Q_{m}$), curves cross (see (13)). Madametologically, integrating over the troposphere ($Y_{c}$, etc.) and neglecting the surface sensible and latent heat fluxes, the surface precipitation flux $F_{p}(O)$ is given by integrals of the convective source terms. For notational brevity, we use height rather than pressure co-ordinates and do not include density, which varies with height.

$$L_{p}(\theta_{e}) \sim \int Q_{c} + Q_{m} \text{d} \theta_{e}$$

(12)

where $\theta_{e}$ is a level in the atmosphere, where the convective source terms are small.

There is a net upward transport of $\theta_{e}$ and moist static energy $h$ associated with this mode since $Q_{c} < Q_{m}$ at low levels, given by

$$\left(\frac{\rho}{\rho_{c}}\right)_{p} (\theta_{e} - \theta_{p}) \sim F_{p} \sim \int \left(\frac{\rho}{\rho_{c}}\right)_{p} h \text{d} \theta_{e}$$

(13)

Arakawa and Chen [56] describe their third mode, as one which increases the separation of the $Q_{c}$, $Q_{m}$ peaks in Figure 5a. This Mode 3 has been drawn on Figure 5a as a modulation of Mode 1, which increases the mid-tropospheric $Q_{c}$ flux, while having little impact on net precipitation. Thus this key diagnostic study shows that the upward $\theta_{e}$ flux is not uniquely coupled to the precipitation. Conceptually one might perhaps associate a larger upward $\theta_{e}$ flux with a dynamical structure which feeds more low $\theta_{e}$ air into the system in mid-levels.

In the feedback to the larger scale, the net precipitation is important, because it is associated with heating and a deep tropospheric ascent mode. The importance of the upward $\theta_{e}$ flux is that this lowers boundary layer $\theta_{e}$ [13][16] [22], and increases the ocean
surface $\Phi$ flux (primarily the moisture fluxes are involved). The importance of this process in regulating convection is discussed in detail by Raymond [58] [59].

The mode 2 in Figure 5b is described by [56] as the component representing deviations of "large-scale" condensation and evaporation, since $(\tilde{Q}_2, \tilde{Q}_2)$ for this mode. Note that, as we have written in it, it represents a condensation over evaporation couplet with no net precipitation at zero $\Phi$ flux (from Equations (1) and (4)). This dimensionally derived mode 2 comes from the variable precipitation component. The mesoscale convective roll. The key consequence of this heating over cooling couplet (which in reality involves condensation and freezing over evaporation and melting) is to force a 2-cell vertical structure over the desert, and a larger-scale convergence in mid-levels near the freezing level as seen in Figure 4.

In the light of this diagnostic study, which is consistent with our GATE September 2 GATE example, and those shown in Johnson [34], I propose that the minimum requirements for a convective parameterization scheme is whether it can represent these 3 idealized modes correctly.

a) A deep convective precipitating mode with an upward $\Phi$ flux, not uniquely coupled to the precipitation (Model 1 and 3)

b) A heating/cooling couplet with no net precipitation and no $\Phi$ flux (Mode 2).

A scheme would then need sufficient closures to be able to determine the magnitude of the net precipitation, $\Phi$ flux, and the heating over cooling couplet, and preferably their time evolution for an evolving unresolved mesoscale convective system.

One immediate question is does the convective Mode 2 have to be parameterized at all? If it is "large-scale" precipitation, why can't the grid-scale processes handle it? (provided there is an adequate prognostic cloud-scheme being fed liquid and solid precipitation from the convective schemes?) We are approximating the core of the so-called scale-interaction problem. What scales are well represented by the large-scale mode? It is clearly unreasonable to expect a two-dimensional geostrophic model with a horizontal grid of 250 km to represent the mesoscale at all, but can a prognostic model with a 50 km grid develop a crude representation of a mesoscale convective roll? The key test 1 would propose is whether the mid-level convergence, shown in Figure 4, develops in tropical convective systems in the model. If not, I would argue it should be forced by parameterically representing the Mode 2 couplet.

In section 4.3, a simple formulation will be proposed.

From an observational perspective, what we see in the GATE data is that, while convective bands initially develop in favorable regions of large-scale waves, and therefore might be regarded as a response to large-scale destabilization, the subsequent convective and mesoscale developments control the evolution of the mass field. In nature, all the scales interact dynamically and can evolve together; but the convective and mesoscale have shorter time scales than the certainly the rotational large-scale fields (although the convective is quite tightly coupled to the divergent field). Because we only simulate the dynamics of the large-scale in our global models, the faster processes must be parameterized. Just as a convection scheme, by introducing precipitation before saturation on the grid-scale is reached, can change the phasing of large-scale dynamical development, so if we introduce a parameterized mesoscale couplet forcing, too will feed back on the large-scale model dynamics sooner, than if we wait for grid-scale processes to reach saturation. Since we know that this inflow at the freezing level is dynamically important in the tropics, it is likely that the impact of this Mode 2 parameterization will be significant.

1.6. MASS FLUX REPRESENTATION OF DEEP CONVECTIVE UPHRAFTS AND DOWNDRAFTS

There have been many discussions of the mass flux representation of cumulus transports following Yau et al. [12] including papers by Johnson [60], Cho [61], Nitza [62], McFarlane [63], Arakawa and Chen [56]. Cheng [55, 16] and Cheng and Yanai [77] and many others. For this reason it is only a constrained summary will be given to illustrate the key issues. The subgrid-scale heating and drying by convective updrafts and downdrafts can be written in bulk form as

$$Q_2 \frac{dP}{dz} \frac{d}{dz} \left[ Q_2 \frac{d}{dz} \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] = \rho C_{a2} \frac{dP}{dz} \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)\frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2}$$

and

$$q_2 \frac{dP}{dz} \frac{d}{dz} \left[ q_2 \frac{d}{dz} \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] = \rho C_{a2} \frac{dP}{dz} \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)\frac{\partial^2}{\partial x^2} \frac{\partial^2}{\partial y^2}$$

We define the terms as follows. The updraft and downdraft mass fluxes are $M_u$, $M_d$ respectively (both defined as positive), and they satisfy mass conservation equations

$$\frac{dM}{dz} + q = 0,$$

$$\frac{dM}{dz} + q = 0$$

where $\rho$, $\beta$ represent entrainment and detrainment rates. The bulk properties of the updraft are her liquid static energy $e_u$, and total water $q_u$, the downdraft is assumed to have no cloud water, so its properties are $e_d$, $q_d$. The environmental fields $\beta_d$, $\beta_d$ is also assumed unstratified. We ignore large-scale horizontal advection of condensate.

$F_{\alpha}(x)$ is the flux of precipitation, which is related to 3 terms

$$F_{\alpha}(x) = \rho C_{\alpha2} \frac{dP}{dz} \frac{d}{dz} \left[ \rho C_{\alpha2} \frac{d}{dz} \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right]$$

where $C_{\alpha}$ is the fallout of precipitation from the updraft, and $F_{\alpha}$ is the evaporation of falling precipitation into the downdraft. Following Arakawa and Chen [56], and the schematic in Figure 5b, we include, in addition to the convective terms, a mesoscale condensation/evaporation couplet term, $Q_{\alpha}$ (which is considered separately from the convective mass flux circulation), and which has both zero west static energy (4) flux and no net precipitation flux. Then $Q_{\alpha}$ satisfies the constraints

$$Q_{\alpha} = Q_{\alpha drip} - Q_{\alpha drip}$$

and

$$\frac{dQ_{\alpha}}{dt} = 0$$

In (14) and (15) we have also not included for brevity the surface sensible and latent heat fluxes or any representation of "turbulent" boundary layer fluxes. From energy conservation in the updraft, which entrains and detrains, and condenses precipitation as it ascends, one can write the updraft budget equation.
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\[ \frac{\partial (M_x u_x)}{\partial t} + \nabla \cdot (M_x u_x u_x) + \nabla \cdot (\phi_x \rho \nabla a) = 0 \]  

(20)

Similarly for the downdraft:

\[ \frac{\partial (M_y u_y)}{\partial t} + \nabla \cdot (M_y u_y u_y) + \nabla \cdot (\phi_y \rho \nabla a) = 0 \]  

(21)

where, in both (20) and (21), it is assumed that air entrained into both updraft and downdraft has the properties of the mean environment. Substituting (20) and (21) in (14) and (15), using (16), (17), and (18), and rearranging:

\[ \nabla \cdot (M_x \nabla a) + \phi_x \rho \nabla a + \phi_x \rho \nabla \cdot (M_x u_x u_x) = Q_M \]  

(22)

\[ \nabla \cdot (M_y \nabla a) + \phi_y \rho \nabla a + \phi_y \rho \nabla \cdot (M_y u_y u_y) = Q_M \]  

(23)

where the net convective mass flux is

\[ M_x = M_{x,0} - M_{x,0} \]  

(24)

The equations (22) and (23) are the heart of many diagnostic studies of convection, and the parametric mass flux representation. Many of the authors cited above have used a spectral representation of the convective transport, but the bulk representation here is sufficient to illustrate the main mechanisms (the entrainment and detrainment terms are poorly known). Note that the convective updraft and downdraft mass fluxes can be formally combined in the leading terms, which have often been described in the literature as "compensating" subsidence terms. They are the terms which represent the bulk heating and drying by deep convection. The physical reality is that the mean grid scale ascent, \( M_{x,0} \), is simply carried upward on small scales as a net transport within convective towers (Rebol and Malkus [64]), that is \( M_{x,0} - M_x \) and the environmental motion between deep convective towers is small.

Cho [61] recognized that if the convective outflows are considered to be at equilibrium, after evaporating any remaining cloud water (note that this depends on the precipitation parameterization), then one can formally drop the detrainment terms in (22) (if virtual temperature effects are neglected), and calculate an \( M_x \) from (also dropping the mesoscale term):

\[ \nabla \cdot (M_x \nabla a) + \phi_x \rho \nabla a + \phi_x \rho \nabla \cdot (M_x u_x u_x) = Q_M \]  

(25)

It could be argued that this is a satisfactory treatment, except near the surface, where cold downdraft outflows cannot sink to equilibrium. However Cho noted, that even if \( M_x \) is calculated from (25), there is no equivalent condition to buoyancy equilibrium in the moisture budget. The moisture content of convective outflow must be determined in (23) to solve the parameterization problem, since it cannot be assumed that \( \phi_x = \phi_y = \phi \). Downdraft outflows are typically unstrained and must be modeled. The precipitation fallout determines the water content of updraft outflows, which, if they are negatively buoyant after evaporating their condensate, will be unstrained after sinking to buoyancy equilibrium [41].

The net vertical mass transport of the deep convective mode is directly related to the net precipitation. Integrating (25) gives

\[ \int \nabla \cdot (M_x \nabla a) + \phi_x \rho \nabla a + \phi_x \rho \nabla \cdot (M_x u_x u_x) \, dx = \phi_x \rho \nabla \cdot \int \phi_x \rho \nabla \cdot (M_x u_x u_x) \, dx \]  

(26)

Substituting (18) in (14) and integrating through the troposphere gives

\[ \int \nabla \cdot (M_x \nabla a) + \phi_x \rho \nabla a + \phi_x \rho \nabla \cdot (M_x u_x u_x) \, dx \]  

(27)

since the convective fluxes dissipate at the integration limits, and the mesoscale term disappears using (19b) if we had not dropped the surface sensible heat flux. It would also be included here. \( F_{x,0}(0) \) is again the surface precipitation flux. Thus (26) and (27) show that the deep mode with net precipitation is formally related to a net deep convective mass flux.

If we add (22) and (23), the mesoscale term, which has no transport because of (19a), again disappears and we get

\[ \nabla \cdot (M_x u_x + M_y u_y) \, dx = \phi_x \rho \nabla \cdot (M_x u_x u_x + M_y u_y u_y) \, dx \]  

(28)

where \( \phi_x = \phi_y = \phi \). In Betts [15], the inflows and outflows were measured directly and the terms in (28), including separate updraft and downdraft mass fluxes were evaluated. As mentioned earlier, at low levels, where \( M_x \) is small, the right hand side is dominated by the outflows of unsaturated downdraft air (since it was assumed that the inflow to the updraft had environmental mean properties, only the outflows appear in (19)). In other diagnostic studies using some networks to derive \( Q_x \), \( Q_y \) (usually calculated), (28) cannot be inverted by itself as it contains 2 unknowns, the updraft and downdraft mass fluxes. Only if downdrafts are neglected, can (28) be immediately inverted, given a cloud model for \( M \), to derive a net convective mass flux \( M (\downarrow) \). In some other diagnostic studies, the ratio of downdraft to updraft mass fluxes has been prescribed (e.g. Johnson [60]) to solve (28). Any two of (22), (23) and (28) can however be regarded as independent. Nitta [62] showed that, if a precipitation parameterization is introduced to determine the remaining cloud water in \( M_x \), in (22), and the downdrafts are assumed to be saturated, then (22) and (28) can be solved simultaneously for both an updraft and downdraft mass fluxes.

A key conclusion is that the mass flux representation of convection is only adequate to the extent that the detrainment terms in (22) and (23) can be either neglected or calculated. In the middle troposphere, the vertical mass flux terms dominate in the downdrafts. As pointed out in Betts [15], near the surface, the low level cooling and drying by convection depends essentially on downdraft outflows. Thus a satisfactory mass flux parameterization for deep convection must calculate both an updraft and a downdraft mass circulation and the properties of the outflows of both the updrafts and downdrafts. All these depend on microphysical and cloud-scale dynamical processes, so a satisfactory general solution has not yet been found.

Convective precipitation is linked to a convective mass flux (see (26) and (27)), but this alone does not determine the upward or downdraft flux, which is also linked to the environmental \( \phi \) structure, and the convective-scale dynamics. Combining (18) and (28), consider the integral to the freezing level \( \tau \) in the mid-troposphere:

\[ (C^T / \phi_y) F_{x,0} + \int F_{x,0} \, dz = \int \phi_x \rho \nabla \cdot (M_x u_x u_x) \, dz \]  

(29)
At the freezing level, $S$ is typically a minimum, so below $z_c$, $\overline{\theta_c}$ is negative, and the first term is positive. The downdraft outflows have typically $\theta_c^-$ negative, so the third term is also positive. If we neglect updraft outflows in the lower troposphere (which means ignoring shallow clouds), it is clear that the strength of mid-tropospheric upward deep convective flux of $k$ (and $\theta_c$) is related to the net convective mass flux, the value of the $\theta_c$ minimum, and the strength and properties of the downdrafts. Since downdrafts bring down mid-tropospheric low $\theta_c$ air, a low value of mid-tropospheric $\theta_c$ contributes in both terms to a larger upward $k$ flux. It is this upward flux of $k$ or $\theta_c$ which plays a key role at the convective interface with the surface fluxes as discussed earlier (see also Raymond [59]).

### 3.7 Diagnostic Retrieval of Mesoscale Source Terms

Arakawa and Chen [56] and Cheng and Yanai [37] discussed a method of extracting the mesoscale flux information from (22) and (23). They defined a parameter $H$, as follows, so as to eliminate the convective mass flux $M_c$:

$$
\frac{H}{(L^0 I q_{0c})} = \frac{Q_k}{\overline{\theta_c}} = \frac{(L^0 I q_{0c})}{\overline{\theta_c}} = \frac{M_c}{\overline{\theta_c}} = \frac{Q_k}{\overline{\theta_c}}
$$

(30)

Cheng and Yanai argued that the detrainment terms are dominated by the detrainment of water from the updrafts ($\overline{q_{0c}}$), this term makes a negative contribution to $H$. To contrast the mesoscale term can be rearranged as

$$
\frac{Q_k}{\overline{\theta_c}} = \frac{Q_k}{\overline{\theta_c}} \frac{(L^0 I q_{0c})}{(L^0 I q_{0c})} = \frac{Q_k}{(L^0 I q_{0c})}
$$

(31)

Since the typical mesoscale convective scale of $Q_k$, positive in the upper troposphere where $\overline{\theta_c}$ is positive, and negative in the lower troposphere where $\overline{\theta_c}$ is negative, this mesoscale term in positive at all levels. Cheng and Yanai [37] noticed that $H$ was positive during GATE convective clusters episodes, so the mesoscale term must dominate over the negative convective scale detrainment term in these cases. They used a cloud model to calculate the detrainment term for mesoscale heating/cooling cases. Figures 6a and 6d from Cheng and Yanai [37] show the results of their diagnostic model at 1800 UTC for the GATE Case Study on Day 245, which we discussed earlier in section 2.3. Figure 6a shows a cross-section of what they called the mass flux in the cumulus environment.

$$
\overline{\theta_c} - \overline{M_c}
$$

(32)

where $\overline{\theta_c}$ is the observed mean vertical mass flux and $M_c$ is the net cumulus mass (updrafts and downdrafts) diagnosed by their model. In the presence of cloud clusters, the authors regarded $H$ as a measure of the mesoscale mass flux. Figure 6b is the corresponding cross-section of the mesoscale convective heating term $Q_k$, found by subtracting the convective contribution (diagnosed by their model) from the total $(\overline{Q_k})_0$, diagnosed from the model budget analysis. The characteristic heating over cooling couple can be seen. Their longitude cross-section at 1800 UTC is along 85.5°N through the center of the GATE array. Note that Figure 6a shows mesoscale ascent over descent at 1800 UTC, while the total mass flux at that time in Figure 3 (at 22°W) does not show descent in the lower troposphere. The diagnostic analysis leading to Figure 6a has however removed the convective mass flux. Figure 3 only shows the ascent over descent pattern later in the lifecycle of the convective system, presumably after the convective circulations have decayed further. It would be useful if the analyses of [37] could be repeated using other cloud models, since it is likely that these general conclusions do not apply to cloud model dependence.

### 4. The Betts-Miller Scheme

#### 4.1 Brief Review

I will not outline the details of the Betts-Miller scheme here, as they are adequately covered in the recent review by [5], but the underlying concepts will be mentioned. It was clear in the decade after the GATE experiment, one of whose key objectives was to resolve the parameterization and 'scale-interaction' problem [1], that we had not found a simple solution. Some developed more detailed cloud models [2] with hierarchies of convective and mesoscale updrafts and downdrafts, but it was clear that the key issue of closure, the linking of all the submodels to each other and to large-scale parameters, was unresolved. The Betts-Miller scheme ([8],[34],[35]) was one response to this. It is an attempt to formulate the convective forcing in a very simple mathematical way, so that perhaps the coupling can be explored in some detail. The idea was lagged convective adjustment towards convective equilibrium profiles of $q$ and $T$. Since we see convection in the tropics adjust the atmosphere towards quasi-equilibrium structures, we can not directly model this process, perhaps more easily than trying to get it as an outcome of complex convection sub-models?

I introduced three concepts:

1. **The moist virtual adiabat**: (the reversible adiabat), rather than the pseudoadiabat, as a reference adiabat up to the freezing level. This was an inference from observations. The
scheme assigns towards a thermal reference profile, which has a specified instability in the lower troposphere, defined with reference to this most vertical adiabat.

b) The freezing level was built into the parameterization in calculating the quasiequilibrium reference profiles, because observationally it appeared to be significant. This was a recognition that the freezing-melting process also plays a role in determining the characteristic thermal structure with a $\theta_e$ minimum.

c) The adjustment was large, to represent the response time of the convective and mesoscale to changes on the large-scale. This gives a smoothed convective feedback, which seems physically more realistic than the off-on behavior of instantaneous convective schemes. In addition the mathematical structure allows the possibility of simplified analytical solutions (Neelin and Yu [65]). Indeed versions of this scheme have done well proof useful in simple tropical climate models (Sen Gupta and Zeman [66] [67]).

The key idea in this parameterization is that, while convection is occurring, the atmosphere is never allowed to get too far from the type of thermodynamic structures we observe. Even if we cannot adequately model the convective terms in detail, if we can construct a model in this way in the face of large-scale forcing, we are imposing the convective sources of heat and moisture, that we would derive by diagnostic methods.

In our later paper (Betts and Miller [6]), we introduced a unstructured model downslope circulation with its own adjustment time, based on a simple coupling of the evaporation into the downslope to the net precipitation. This was an attempt to directly model unstructured downslope outflows into the boundary layer, and some improvements in the tropical climate resulted [68], Sturgis et al [69]. In [70], the adjustment near the surface was not well constrained.

Even a decade after its introduction, this lagged adjustment approach still has validity, as we have yet to agree on the key questions of what determine the dynamics and thermodynamic transports on all the unresolved scales (from individual cells to say the 50 km scale) to the scales resolved by global models. Two further extensions will now be proposed.

4.2 ADJUSTMENT TIME SCALES FOR THE BETTS-MILLER SCHEME

Betts and Miller [6] present no theory for determining their convective adjustment time ($\tau$), other than the empirical approach of setting $\tau$ short enough to make generation on the grid scale infrequent. They recognized that this required smaller $\tau$ as higher resolution models to maximum values of grid-scale increase. A simple dynamical basis for $\tau$ is proposed here. Elsewhere in this volume, Shipman [20] discusses the critical gravity wave associated with the convection feedback to the large-scale. The first, a vertical wave-mode 1, spanning the deep troposphere has the fastest gravity wave propagation speed of 50 $\text{m} \cdot \text{s}^{-1}$. While the second wave-mode 2, the dipole with a node at the freezing level, has a slower gravity wave speed $\sim 25 \text{ m} \cdot \text{s}^{-1}$. These are the two convective modes which interact rapidly with the large-scale. These are the also the mode 1 for which the diagnostic study of Arakawa and Chen [56] showed characteristic structure and transport. Using the phase speed associated with these modes, one can calculate corresponding adjustment times for the Betts-Miller scheme. Table 1 shows values for three model grid lengths 60 km, characteristic of the ECMWF forecast model (with spectral truncation of T-213), 120 km and 400 km, characteristic of a relatively low resolution climate model. This would give a formal basis for changing $\tau$ with horizontal resolution, although again some empirical adjustment may be necessary.

The adjustment time scales suggested by Betts and Miller [6] (1 hr at 1065, corresponding roughly to 120 km horizontal resolution) lie between the values given for the fast Mode 1 and the slower Mode 2, suggesting that their empirical approach had some merit.

<p>| Table 1: Adjustment times as a function of horizontal scale for two primary wave-modes |
|-----------------------------------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Wave-mode</th>
<th>Wave Mode 1 (Deep Troposphere)</th>
<th>Wave Mode 2 (Waves at Border of Turbulent Layer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed</td>
<td>(\text{m} \cdot \text{s}^{-1})</td>
<td>(\text{m} \cdot \text{s}^{-1})</td>
</tr>
<tr>
<td>(km)</td>
<td>(60 km)</td>
<td>(120 km)</td>
</tr>
<tr>
<td>Wave Mode 1</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Wave Mode 2</td>
<td>25</td>
<td>20</td>
</tr>
</tbody>
</table>

However this heuristic dynamical model for $\tau$ does raise clearly a new issue. Gravity wave propagation can adjust quickly the thermal (temperature) structure on larger scales with these adjustment times. How, however, does the moist structure is adjusted on the same time scale, as is assumed in Betts and Miller [6]. This relates back to the discussion in section 3.6. Vertical displacements by gravity waves can simulate the thermal mass transport model represented by equation (25), but in the equivalent moisture equation the "dampening terms" cannot be neglected. This needs further study.

One unanswered question, which was raised in section 3.5, is whether convection parameterization schemes should have an explicit formulation of this wave-mode 2 forcing of the large-scale. One could argue that a global model with 60 km resolution, and an explicit cloud scheme, might possibly generate a marginal, but adequate representation of the transform precipitation and this mesoscale couplet on the grid-scale. However without the non-hydrostatic mesoscale dynamics, it may not. This could be explored numerically since the key feature, the development of the strong mid-lower inflow in the mature and decaying phase of a mesoscale system, should be visible. Here we will argue the reverse: namely that only by improving the representation of a mesoscale wave-mode 2 couplet, linked to the convective scale, will the convective forcing of the large-scale be adequately represented in most, if not all, hydrostatic global models. In any event, this is a key scale separation issue, which needs exploring by a variety of techniques.

4.3 EXPLICIT PARAMETERIZATION OF MESOSCALE CONDENSATION- EVAPORATION COUPLET

We will illustrate this suggestion by proposing a simple extension to the Betts-Miller scheme (or any other convection scheme) to include a mesoscale precipitation-evaporation couplet. This is a further extension of Betts and Miller [6], who proposed a formulation for an unamortized downslope circulation, with its own time-scale linked to the precipitation. This idealized Mode 2 couplet has no $\theta_e$ transport, and no net precipitation, but does redistribute entropy by means of condensation/evaporation shift above evaporation/melting basin. In the tropics the freezing level is almost half the pressure depth of the
conductive layer, so we can then represent the convective mesoscale forcing mechanisms shown schematically in Figure 3b, as a simple time function in pressure coordinates:

\[ Q_p = \frac{\partial \nu}{\partial t} + \frac{\partial (\nu P_v)}{\partial \eta} = \sin(2\pi \nu \eta) \frac{\partial P_v}{\partial \eta} \]  

(33)

where \( \nu \) is the surface pressure, and \( P_v \) is the top of the deep convective layer. This satisfies the constraints (19a) and (19b) (in pressure coordinates). We need to link this exchange to the net convective heating, so the simple closure is proposed

\[ F_p = \beta F_P \]  

(34)

where \( F_P \) is the surface precipitation rate, and the tunable parameter \( \beta \) is perhaps \( \approx 0.2 \) (Johnston [14]). The magnitude of \( \beta \) effectively couples the adjustment time and amplitude of this mode to that of the deep convective mode. This mesoscale mode parameterization would be imposed in addition to the convective parameterization. It does not affect the precipitation or vertical \( \theta_e \) transport, but it does change the vertical redistribution of enthalpy and water. The sense of the lifecycle of mesoscale systems has still not been addressed. In climate models in which the resolved horizontal scale is larger, one might plausibly argue that within one model grid cell, large enough to include several convective mesosystems, that the simultaneous representation of convective and mesoscale components is adequate. In higher resolution forecast models, this may not be satisfactory, and some means of representing convective system lifecycles may still be needed. Indeed, since the evolution of many of the GATE mesoscale systems also appeared to be linked to the diurnal cycle, further work on this is needed.

5. Summary

This paper summarizes some important concepts in the parameterization of shallow and deep convection in large-scale numerical models. Starting in the early 1970s, diagnostic models have influenced the development of cumulus parameterizations, and field programs in the tropics, such as VMB/TEX and GATE, showed the complexity of the life-cycle evolution of mesoscale systems with convective and mesoscale updrafts and downdrafts, coupled to the condensation and evaporation process. It is important to appreciate the thermodynamic differences between nonprecipitating and precipitating convection. Shallow, non-precipitating convection can be modeled using a mass transport model, and any two parameters independent conserved variables (or alternatively using convective adjustment to quasi-equilibrium convection structures). Irrationally, despite its simplicity and its key role in controlling the surface fluxes both over the ocean and over land, we have yet to parameterize shallow convection satisfactorily in numerical models. In contrast, precipitating convection is much more complex and difficult to parameterize. Precipitation falls from updrafts and evaporates driving downdrafts, so \( \theta_e \) is not conserved. Only the \( \theta_e \) flux depends on the updraft and downdraft mass fluxes. Cloud microphysics controls precipitation, and in addition the downdraft thermodynamics is poorly known, as downdrafts remain unsaturated, with a substructure that depends on small-scale dynamical and microphysical balances. Consequently the \( \theta_e \) flux is not tightly coupled to the enthalpy and water fluxes, which depend greatly on the precipitation flux, and its change with height through condensation and evaporation of falling precipitation. Diagnostic studies have shown that while the principal deep convective mode is tied to precipitation, and a deep tropospheric upward mass circulation, the upward \( \theta_e \) flux, which interacts strongly with the subcloud layer over the oceans, is not uniquely coupled to the precipitation. In addition, diagnostic studies show there is a key second convective mode associated with the mesoscale antiloo coupled of ascent over descent, which might be parameterized in the same way as large-scale convection, as a condensation-evaporation coupling without any \( \theta_e \) transport or surface precipitation.

The concepts behind the Betts-Miller scheme are briefly reviewed, and two extensions are proposed. One is a theoretical basis for the adjustment time scale, based on the gravity wave propagation speed of the two primary modes. The other is a suggestion that the mesoscale convective mode be explicitly parameterized, and coupled to the surface precipitation. Although this does not address the cloud cluster life cycle issue, which may matter in high resolution global models, or through coupling to the diurnal cycle, it might provide new insight into how these two convective modes interact with the large-scale flow.

6. Acknowledgments

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Appendix. Relation between relative humidity and \( P \)

The relation between saturation pressure difference \( P \) and RH is of some interest, because \( P \) is more directly related than RH to vertical circulations and convective processes in the atmosphere (see 3.3). Consider an unsaturated parcel at \( (T, q) \) with vapor pressure \( e \), saturation vapor pressure \( e^* \) lifted adiabatically to its saturation pressure \( P \). Consider the linearization between \( p \) and \( p' \), along the dry adiabat \( \Theta \) and line of constant mixing ratio \( q \), shown in Figure A1.

\[
\begin{align*}
\frac{\partial e^*}{\partial q} &= \frac{\partial P}{\partial q} \\
\frac{\partial P}{\partial q} &= \frac{\partial P}{\partial e^*} \\
\frac{\partial \Theta}{\partial q} &= \frac{\partial P}{\partial \Theta} \\
\frac{\partial \Theta}{\partial q} &= \frac{\partial P}{\partial \Theta}
\end{align*}
\]  

(A1)

Figure A1. Schematic thermodynamic diagram showing paths of temperature, dewpoint and the corresponding vapor pressure of an unsaturated parcel lifted to saturation.

\[
(\Theta, q) = \frac{\partial \Theta}{\partial q} \quad (\Theta, e^*) = \frac{\partial \Theta}{\partial e^*} \\
(\Theta, p) = \frac{\partial \Theta}{\partial p} \\
(q, p) = \frac{\partial q}{\partial p}
\]

(A2)

On the dry adiabat, \( e^* \) changes rapidly. Expanding and using Clapeyron gives

\[
\begin{align*}
\frac{\partial e^*}{\partial p} &= \frac{\partial P}{\partial p} \\
\frac{\partial P}{\partial e^*} &= \frac{\partial P}{\partial e^*} \quad (\Theta, e^*) \\
\frac{\partial \Theta}{\partial e^*} &= \frac{\partial \Theta}{\partial e^*} \quad (\Theta, e^*) \\
\frac{\partial \Theta}{\partial e^*} &= \frac{\partial \Theta}{\partial e^*} \\
\end{align*}
\]

(A3)

Although in A3, \( (\Theta, e^*) \) varies strongly from \( p \) to \( p' \) on constant \( \Theta \), we can linearize between the saturation point (where \( e^*(P) \), using (A2)) and \( e^*(p) \) at pressure \( p \), and to good approximation also neglect the variation of \( \Delta \). Doing this, A1 simplifies to

\[
(\Theta, e^*) = \frac{\partial \Theta}{\partial e^*} \quad (\Theta, e^*) = \frac{\partial \Theta}{\partial e^*} \\
(\Theta, e^*) = \frac{\partial \Theta}{\partial e^*} \quad (\Theta, e^*) = \frac{\partial \Theta}{\partial e^*}
\]

(A4)

Defining \( RH = e/e^*, \) and 2 \( \frac{\partial e}{\partial e^*} = \frac{\partial \Theta}{\partial e^*} \), gives, on rearrangement, equation (6)

\[
P = \rho \frac{\partial P}{\partial \rho} \frac{\partial \Theta}{\partial \rho} \frac{\partial \rho}{\partial \rho} \rho \frac{\partial \rho}{\partial \rho}
\]

(6)

The thermodynamic coefficient, \( A \), increases with decreasing temperature from 2.6 at 25°C to 3.4 at -40°C. (6) is quite accurate: it gives \( P \) to ±1 mb for RH=0.5 and \( p < 200 \) mb.