551.511.33:551.558.1:551.576.1

Reply to comment on the paper 'Non-precipitating cumulus convection and its parameterization'

By A. K. Betts

I agree with this interpretation (Deardorff *et al.* 1974) of the laboratory measurements of Deardorff *et al.* (1969). The authors have pointed out an important distinction between actual measurement of the downward heat flux below an inversion, and that heat flux which is implied in a model when a stable layer is 'mixed' to constant potential temperature. However I still think that, in the atmosphere, the actual downward $\overline{w'\theta'}$ is not so small as reported in Deardorff *et al.* (1969). Since atmospheric tests of this type of model are intended in the approaching GATE, I think this needs further discussion.

As in Deardorff *et al.* (1969), the observed stratification is the observational clue to the magnitude of the downward $\overline{w'\theta'}$ just below the top of the mixed layer. This can be illustrated by considering an inversion layer of finite thickness instead of the limiting case of infinitesimal depth (Betts 1970). Extending Deardorff's present notation, one may write budget equations (neglecting radiation and assuming no mean vertical motion) for the two layers: surface to h_1, h_1 to h_2 (see Fig. 1).

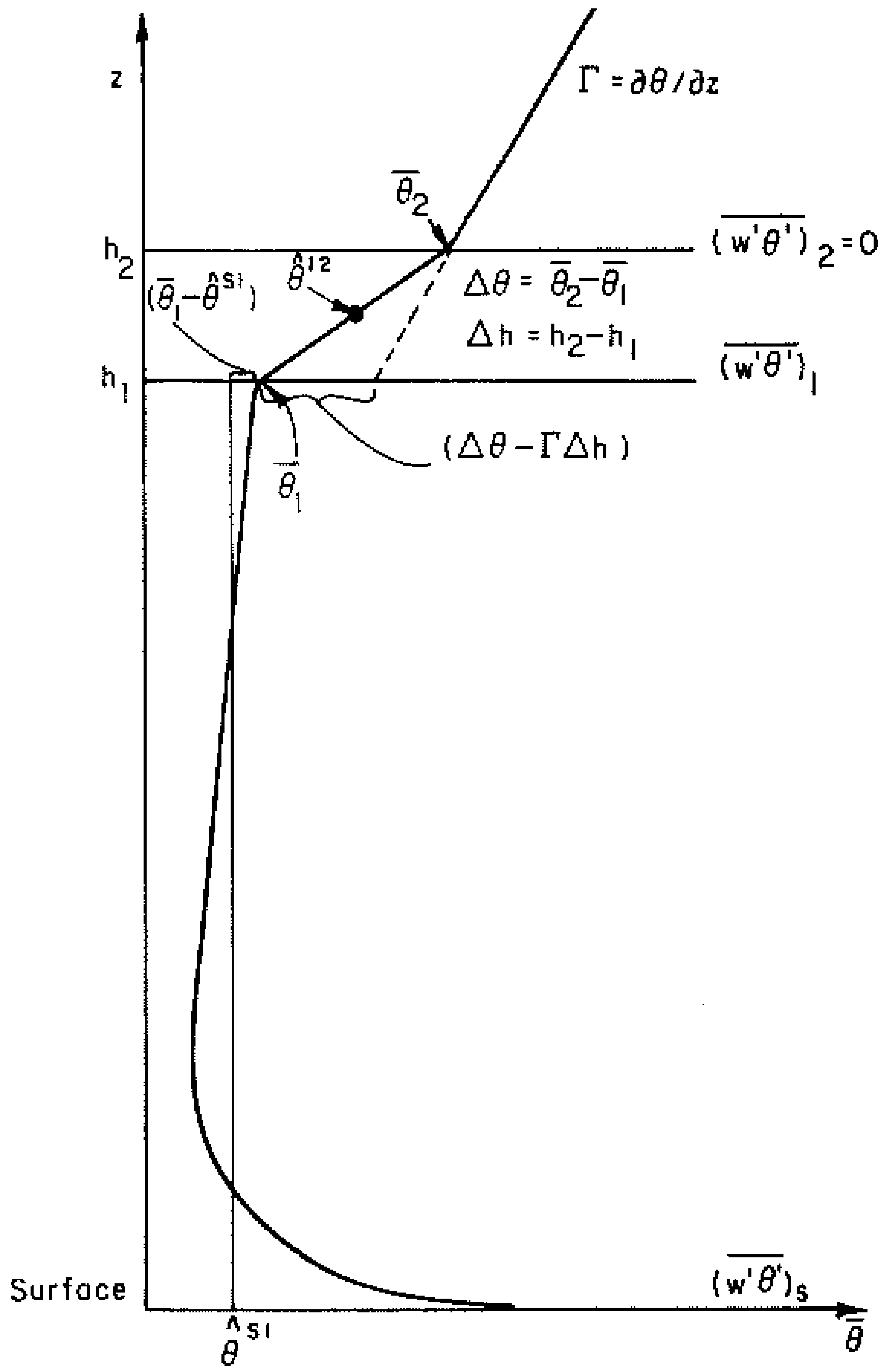

Figure 1. Sketch profile of potential temperature for a layer of dry convection, topped by an inversion layer.

$$\frac{d}{dt}[h_1\hat{\theta}^{s1}] = (\overline{w'\theta'}_s - \overline{w'\theta'}_1) + \frac{dh_1}{dt}\bar{\theta}_1 \quad . \quad . \quad . \quad (1)$$

$$\frac{d}{dt}[(h_2 - h_1)\hat{\theta}^{12}] = \overline{w'\theta'}_1 + \frac{dh_2}{dt}\bar{\theta}_2 - \frac{dh_1}{dt}\bar{\theta}_1 \quad . \quad . \quad . \quad (2)$$

The level h_2 has been chosen as the level where $\overline{w'\theta'} = 0$ and $\hat{\theta}^{s1}$, $\hat{\theta}^{12}$ are averages for the two layers. If we consider the simple case, given by

$$h_2 - h_1 = \Delta h = \text{constant}$$

then Eq. (2) reduces to

$$\overline{w'\theta'}_1 = -\frac{dh_1}{dt}\Delta\theta + \Delta h\frac{d\hat{\theta}^{12}}{dt} \quad . \quad . \quad . \quad . \quad (3)$$

where $\Delta\theta = \bar{\theta}_2 - \bar{\theta}_1$. If, further, $\Delta\theta$ is constant,

$$\frac{d\hat{\theta}^{12}}{dt} = \Gamma\frac{dh_2}{dt} = \Gamma\frac{dh_1}{dt} \quad . \quad . \quad . \quad . \quad (4)$$

where Γ is $\partial\bar{\theta}/\partial z$ above h_2. Eq. (3) then becomes

$$\overline{w'\theta'}_1 = \frac{dh_1}{dt}(\Gamma\Delta h - \Delta\theta) \quad . \quad . \quad . \quad . \quad (5)$$

Thus the flux at h_1 is downward if

$$\frac{\Delta\theta}{\Delta h} > \Gamma$$

that is, if the layer h_1 to h_2 is more stable than the layer above. Thus one can conclude diagnostically that if a stable inversion layer is observed to persist with $\Delta\theta$, Δh near constant as h_1 changes (or does not move with the mean vertical motion), then $\overline{w'\theta'}_1$ at the base of the inversion is downward, and has the value given by Eq. (5). For this case the inversion strength can be defined as

$$\Delta\theta' = \Delta\theta - \Gamma\Delta h$$

The ratio of $\overline{w'\theta'}_1$ to the term $(\bar{\theta}_1 - \hat{\theta}^{s1})\, dh_1/dt$ in Eq. (1) (Eq. (5) of Deardorff *et al.* 1974) is just $\Delta\theta'/(\bar{\theta}_1 - \hat{\theta}^{s1})$, and this can be estimated from a mean profile (see Fig. 2).

How this mean profile should be determined by observation of the atmosphere is not obvious and different methods will lead to different estimates as to the magnitude of $\overline{w'\theta'}_1$. For example, horizontal aircraft flights are likely to generate a smoothed profile of $\bar{\theta}(z)$, and low measurements of $\overline{w'\theta'}_1$ (by gustprobe) simply because of variations in the height of the inversion. An alternative but more selective (possibly, therefore, not representative of a horizontal average) method of determining a mean $\theta(z)$ profile is presented here. Fig. 2 is an average of fourteen soundings from the tropical atmosphere over land during conditions of suppressed convection (small cumulus). Virtual potential temperature, θ_v, has been plotted as being representative of air density. Before averaging, the pressure levels of each sounding were scaled so that the base of the inversion was at $p^* = 1{\cdot}0$ in order to preserve the inversion in the average. The variation of inversion height was only from 911 mb to 919 mb. The layer mean θ_v has been taken from $p^* = 0{\cdot}1$ to $1{\cdot}0$, not on theoretical grounds but because the lowest 10 mb cannot be considered to be an accurate profile. Including the surface layer could increase θ_v by 0·1 deg K. From Fig. 2 one would conclude, if $\Delta\theta$, Δh are not changing with time, that

$$(\overline{w'\theta'_v})_1 = -0{\cdot}55\, dh_1/dt$$

In the mixed layer model, the corresponding total downward model heat flux below the inversion would only be increased to $-0{\cdot}75\, dh_1/dt$. A corresponding value of k, based on Eq. (45) of Betts (1973), is 0·3. Seventy-three per cent (0·55/0·75) of this value of k is associated with $(\overline{w'\theta'_v})_1$.

I conclude that the atmospheric validation of the mixed layer model is not straightforward. With the likelihood of variations in the mixed layer height on many scales 'representative measurements of negative heat flux at $z = h$' (Deardorff *et al.* 1974, conclusion (c)) may be difficult to obtain. The study of profile structure can, however, give indirect estimates of $\overline{w'\theta'}_1$, and this does not appear to be small in the atmosphere.

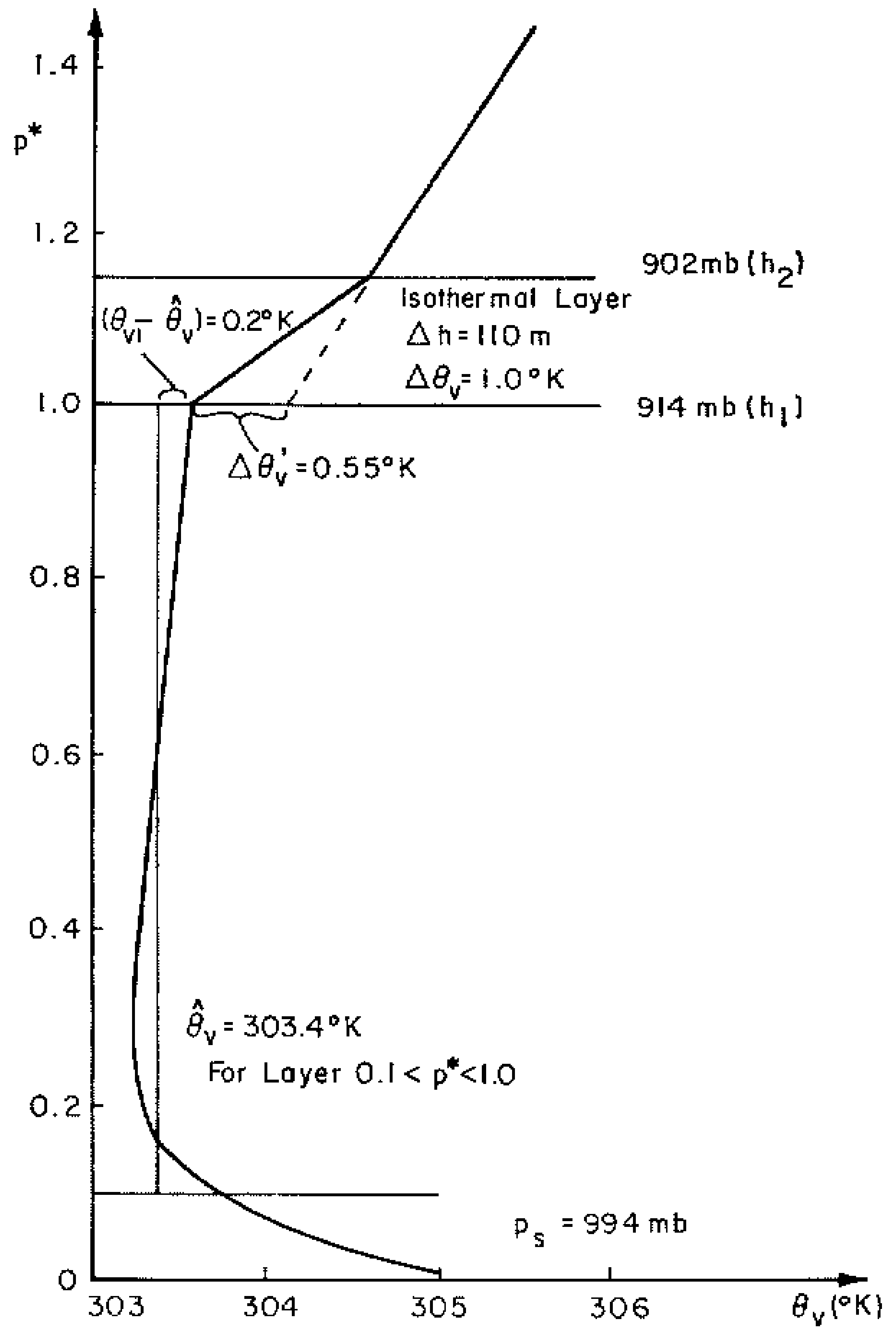

Figure 2. An observed vertical profile of mean virtual potential temperature (θ_v) against scaled pressure (p^*) through the layer of dry convection.

References

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Acknowledgment

This work has been supported by the Atmospheric Sciences Section of the National Science Foundation under Grant GA–36323.

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28 February 1974