# A Relationship Between Stratification, Cloud Depth, and Permitted Cloud Radii

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11 December 1972

## ABSTRACT

A one-dimensional cumulus model is used to interrelate cloud radius, stratification and cloud height. It is suggested that if only certain ranges of cloud height and radius are permitted, then from the stratification, one can predict, using a model, allowed cloud radii and a corresponding depth to the convective layer.

## 1. Introduction

A major objective of the GATE experiment (GATE, 1972) is to extend our knowledge of convection in the tropics, so as to improve the convective parameterization schemes being developed for numerical forecast models. The relationship between the cloud fields, and the larger scale meteorological fields of temperature, humidity and wind, remains largely unresolved. One obstacle to convective parameterization might be regarded as the specification of the depth of the convective layer in terms of large-scale variables. Simple one-dimensional cloud models have been developed which predict the cloud top for a cloud of given size (radius) in a known temperature and water vapor stratification (Simpson *et al.*, 1965), but these models do not suggest how cloud size may be determined. Other theoretical models have predicted that some relationship should exist between cloud vertical and horizontal extent (Kuo, 1965; Asai, 1967), but these are not based on realistic atmospheric stratifications, nor do they account well for the entrainment process, known to be important in determining cloud height.

The purpose of this note is to suggest that this problem may be closed, and to outline a simple framework with which to interpret observations of clouds and atmospheric thermodynamic structure, in such a way as to provide a depth-radius relationship, which can be used in convective parameterization schemes. This closure, depicted schematically in Fig. 1, follows if some permitted radius/depth ($R/H$) relationship exists. If such a relationship can be deduced from atmospheric observations (which may prove easier than from numerical integrations of three-dimensional cloud models), then by an iterative procedure, consistent values of $R$ and $H$ can be found using any cloud model. It is probable that a range of permitted "cloud radii" will exist, although clearly the simpler the spectrum of cloud sizes, the simpler parameterization becomes.

At the same time, the use of a model to compute $H$ as a function of $R$ is a sensitive method of classifying atmospheric stratifications. These may be regarded as characteristic of the cloud population (Ludlam, 1966), and could be used in themselves in a parameteric scheme. This approach has been used theoretically (Betts, 1973) in a lapse rate model for nonprecipitating cumulus convection. Alternatively, one might attempt to deduce detailed information on the cloud population from the stratification. However, estimation of cloud number density $N(R)$ in, say a grid volume, requires additional information or assumptions about the time dependence of the field.

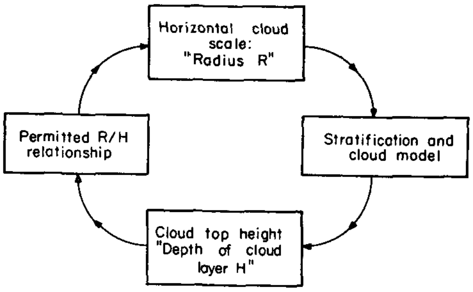

Fig. 1. Schematic prediction of the depth of the convective layer $H$, given the stratification.

## 2. Model calculations

The NOAA EMB-68 series, one-dimensional cloud model (Simpson and Wiggert, 1971), which has been developed by careful observational comparison with atmospheric clouds, was used to illustrate these suggestions. The cloud top was predicted for a spectrum of cloud radii for four mean Florida soundings (from Simpson and Woodley, 1971), with cloud characteristics described as:

- (i) suppressed growth
- (ii) cut-off tower growth
- (iii) explosive growth on seeding
- (iv) large natural growth

These results are presented in Fig. 2 in terms of $H/R$ vs $R$ where $R$ is the model cloud tower radius, and $H$ the cloud depth defined as (peak height—cloud-base height). The four curves are quite separate for middle sized radii, with the ratio $H/R$ increasing at a given $R$ through classes (i)-(iv) as one might expect. If there exists a curve representing a minimum permitted $H/R$ (i.e., by forbidding short, fat clouds), then there is correspondingly, only a range of allowed values of $R$ and $H$ for each mean sounding. Such a lower bound on $H/R$ is hypothesized and shown as a heavy dashed line. This is drawn so that the curve (iv) is wholly above it, while curves (i)-(iii) fall below this bounding line at some $R$ (and correspondingly some $H$).

For curve (iv) all radii are permitted up to sizes large enough to reach the upper troposphere as observed. At the other extreme [curve (i)] only very small clouds are "allowed." Curves (ii) and (iii) rise above the dashed line at larger $R$; this rise results from natural glaciation, which is allowed to occur in this model in

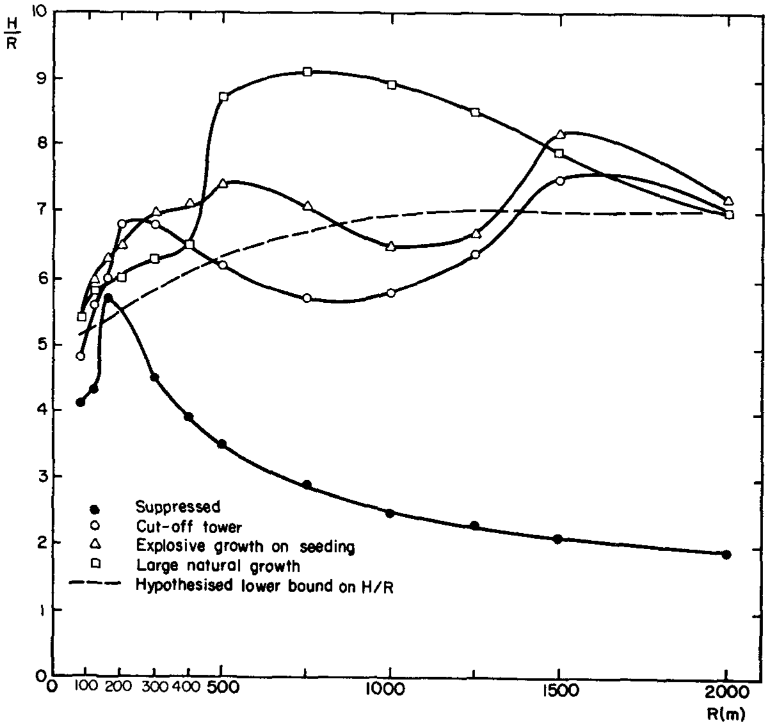

Fig. 2. Plot of computed cloud depth over radius ($H/R$) vs $R$ for four mean Florida soundings. See text for discussion of hypothesized lower bound.

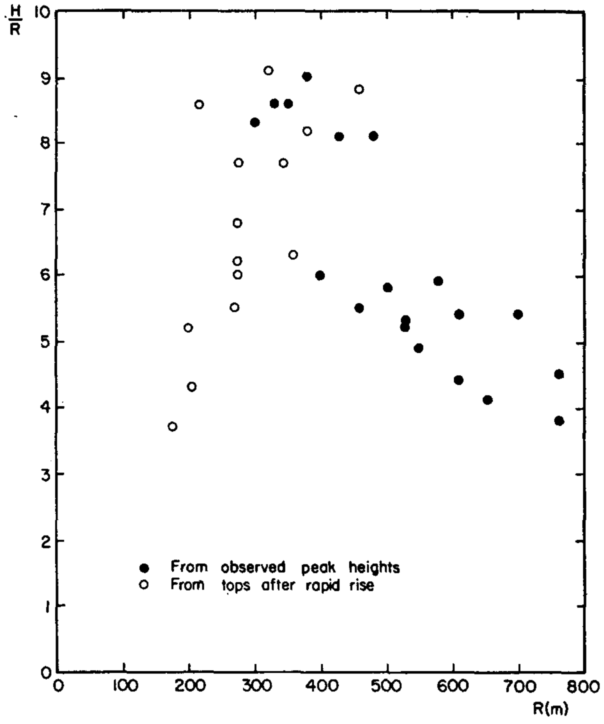

FIG. 3. Some observed values of $H/R$ for individual clouds as a function of $R$.

the temperature range −15 to −40C. However, if one supposes that larger clouds grow from smaller ones, then, in general, larger clouds will not exist if there is a "forbidden band" in $R$. Curve (iii) which only just dips below the dashed curve, is lifted above it if the model cloud is artificially seeded, releasing latent heat at −4 to −8C. Thus, seeded clouds are permitted to grow to the upper troposphere.

The fall of the dashed $H/R$ curve at smaller $R$ is conjecture to permit the existence of small clouds, and might be justified theoretically, since for small clouds, $H$ could be a serious underestimate of the vertical extent of the cloud circulation, if this extends well below cloud base. Curves (ii)–(iv), are not very sensitive to cloud base, taken as 910 m, which is an input to the EMB model. For curve (i), where cumulus clouds are marginal, a higher cloudbase of 1010 m was chosen, consistent with observation: a cloud base of 910 m gives an even lower $H/R$ curve. Clearly, the curves in Fig. 2 are unique to the model used to derive them, but they illustrate that the combination of a model and cloud observations to define a permitted $H/R$ curve is a useful tool in characterizing atmospheric stratifications, and establishing the depth of a convective layer in a numerical model.

## 3. Observational evidence

Observational studies of cloud populations present difficulties. Aerial photography from aircraft (Plank, 1969) or satellites will see clouds in all stages of their life cycle of growth and decay, and a sample of instantaneous values of cloud height and diameter will not give correct statistics on the peak heights reached by clouds of a certain diameter. The studies using ground-based photogrammetry by Saunders (1961, 1965) for the rise of active cumulus towers provide some comparison with the model of the preceding section. Saunders (1961) found values of $H_1/R_1$ (averaged for different days), ranging from 4.8 to 5.7, where $R_1$ is the radius of a bubble and $H_1$ its height above a virtual origin (somewhat below cloud base).

Some other data collected by Saunders in Barbados in 1963, and loaned to the author by Joanne Simpson, are presented in Fig. 3, as $H/R$ vs $R$ for comparison with Fig. 2. There are two classes of points: the open circles represent heights reached by fairly small towers after a period of rapid rise, and are for a single day; the solid points are a wider sample from several days based on peak heights reached by clouds.

These measurements of cloud tower radius are closely comparable to the tower radius $R$ which is an input to the EMB model of Section 2. The scatter in Fig. 3 is large; the open circles have a mean $R$ of 290 m and $H/R$ of 6.7, while the corresponding values for the solid points are 520 m and 6.2. The apparent variation of $H/R$ with $R$ may not be significant.

It would be possible during the GATE experiment to gather a complete set of cloud data on all scales to be analyzed in conjunction with the ship rawinsonde data.

## 4. Limitations

There are two major problems avoided in this analysis. The first is that the cloud model used is a purely thermodynamic model which does not include any interaction between a cloud and the wind-shear field. It is not known how significant this is in determining either atmospheric thermodynamic structure or cloud $H/R$. The approach outlined above could be used, however, to expose the limitations of a purely thermodynamic cloud model by defining the effect of vertical shear, in particular, on cloud vertical growth, and atmospheric stratification. The second problem is that a model of an isolated cloud is not likely to be adequate for an organized cloud field. The stratification characteristic of each mode of organization may have to be identified, until the factors controlling convective organization in the atmosphere are clearly known.

## 5. Conclusion

The need for an observational program as well as more theoretical work on cloud models is very clear. However, an observational program can only test directly those convective models or parameterization schemes which are expressed in terms of observable quantities. This note has attempted to define one method of classifying the thermodynamic stratification, to deduce preferred scales of cumulus convection. This understanding will facilitate the incorporation of convection parametrically in numerical simulations of the atmosphere.

*Acknowledgments.* This work was begun during a visit to the NOAA Experimental Meteorology Laboratory in Miami; the author would like to thank Dr. Joanne Simpson and Dr. V. Wiggert for their help in running computations on the EMB model. This research was supported by the Atmospheric Sciences Section, National Science Foundation, under Grant GA 11637.

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