The Parameterization of Deep Convection and the Betts-Miller scheme

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References

Some history of two decades of diagnostic studies
– Convective mesosystem mass flux life cycle: GATE day 245

Some thermodynamic issues
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– and different convective modes

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– Reference profiles and adjustment timescale
– Addition of downdraft
– Remaining issues

Three convective modes: what needs to be parameterized?

Conclusions

Appendix on "Mass flux formulation of convective updrafts and downdrafts.

References.

** Betts, A.K. and M.J. Miller, 1993. The Betts-Miller scheme. Chapter 9 in *"The Representation of Cumulus Convection in Numerical Models of the Atmosphere"*. (Eds. K.A. Emanuel and D.J. Raymond.) *Amer. Meteor. Soc., Meteor.* Mon. **24**, No 46, 107-121. [Source code for Betts-Miller (1993) scheme is at http://www.met.rdg.ac.uk/~mike/betts_miller.html]

** Betts, A.K.: 1997, 'The Parameterization of deep convection', Chapter 10 (pp 255-279) in *"The Physics and Parameterization of Moist Atmospheric Convection,* Ed. R. K. Smith, NATO ASI Series C: Vol. **505**, Kluwer Academic Publishers, Dordrecht, 498pp

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Betts, A. K., 1985: Mixing line analysis of clouds and cloudy boundary layers. *J. Atmos. Sci.*, **42**, 2751-2763. *[background to shallow scheme]*

GATE-era development of Cumulus Parameterizations

1) *Arakawa and Schubert [1974]* and the diagnostic study of *Yanai et al. [1973]…*
– **Cloud model based on entraining updrafts** -- has dominated this school of cumulus parameterization up to this date.
-- They showed that using this simple cloud mass flux model,
θ_E flux, mass flux and precipitation were coupled
-- Subsequently this model has been **extended to include convective downdrafts** [Cheng, 1989; Cheng and Yanai, 1989; Arakawa and Cheng 1993].

2) *Betts [1973]* used data from VIMHEX to formulate an **updraft-downdraft budget model for mesoscale cumulonimbus systems**.
-- **Using θ_E conservation *separately*** for the updraft and downdraft circulations separates updraft condensation, evaporation into the downdraft and net precipitation.
-- about half the updraft condensation was evaporated into the downdrafts
-- ***upward* mass circulation in the growth phase, and a *downward* circulation in the decay phase**
--low level transformation by deep convection was dominated by cold, dry downdraft outflows.

3) *Gray [1973]* reached similar conclusions from larger scale budgets.

– Other papers on convective and mesoscale downdrafts.
Zipser [1969, 1977], Houze [1977], Miller and Betts, [1977]

GATE Diagnostic Studies

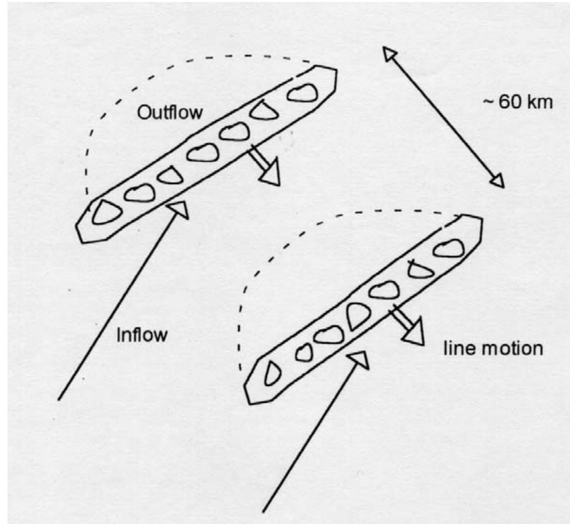
By the 1980's, it was clear that parameterizing tropical convection needed a model for the lifecycle effect of mesoscale complexes, which evolve from lines of cumulonimbus to mesoscale anvil systems within 6-12 hours, on horizontal scales that are not well resolved in global models.

Leary and Houze [1979a,b; 1980].
Barnes and Sieckman [1984].
Esbensen and Ooyama [1983]
Johnson [1984]
Tollerud and Esbensen [1985]
Esbensen et al. [1988]
Cheng [1989a,b] and Cheng and Yanai [1989]

It was also clear from the detailed analyses of Ooyama in the late 1970's, that the **freezing level, which in the tropics is in the mid-troposphere, is dynamically significant**.

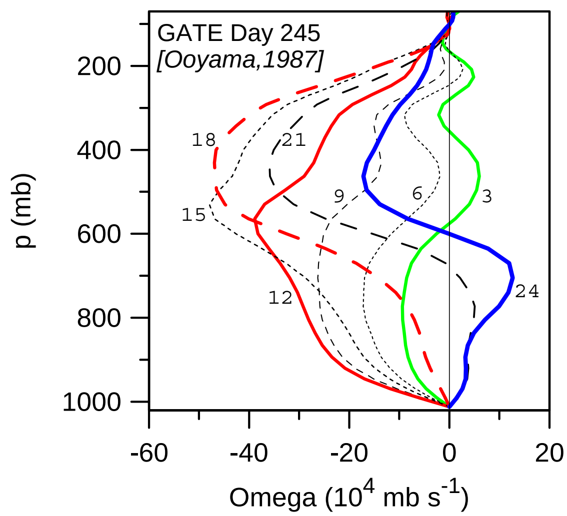
GATE Cloud Cluster Lifecycle on Day 245

Schematic, typical of the GATE environment with strong shear in the low levels. Within the time-span of a typical mesoscale aircraft mission, the observer would see the development of cumulonimbus bands oriented along the low level shear, with inflows on one side and a developing anvil outflow to the rear.



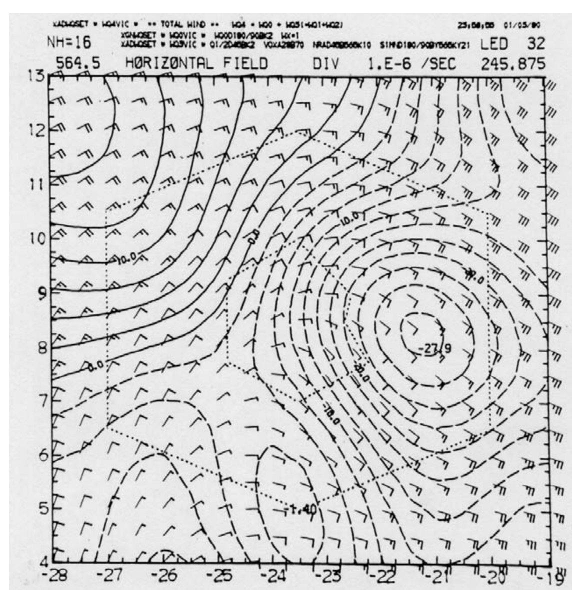
Lifecycle mass flux of a GATE cluster on Day 245 [Sept. 2, 1974]

03UTC [green] low level convergence
12UTC [red] peak ascent mid-trop.
18UTC [red dash] peak at 400mb
24UTC [blue] descent over ascent couplet



Mid-level Convergence in decay phase

At 21UTC mid-tropospheric convergence peaks at 2.8 10^{-5}. [This is bigger than the low level convergence at any time in the lifecycle.].



Thermodynamic issues

NON-PRECIPITATING CONVECTION

As long as ‘cloud’ droplets are small, they are carried with air parcels.

-- Reversible thermodynamics apply provided there is no mixing; and ascent and descent of cloudy parcels follow saturated reversible adiabats
[Betts 1973a; Emanuel 1994].

-- The enthalpy and water transports are coupled to the mass flux, whether this is up or down.

-- **essential irreversibility caused by mixing between cloudy air parcels and their unsaturated and stably stratified environment.** Cloudy parcels originating below cloud base, if they mix with air from above cloud base, which has a higher potential temperature, θ, as well as being unsaturated, cannot descend back as far as they ascended.

TRANSITION TO PRECIPITATION

-- Once cloud droplets grow large enough to fall out of air parcels into unsaturated air, the cloud microphysics becomes important and the entire thermodynamic picture changes.

Precipitating Convection
– and the Limitation of Mass Flux Models

The fraction of the precipitation that falls out is important.

θ_L is no longer conserved in the updrafts and the subsequent thermodynamics of updraft parcels is different, as their cloud water is reduced considerably. Even in small Trade-cumulus clouds, the fallout of precipitation produces layering in the atmospheric structure. [Betts and Albrecht 1987].

For evaporation driven downdrafts, the key thermodynamic issue is that only one air parcel parameter, θ_E, the equivalent potential temperature, is closely conserved. **Downdraft parcel subsaturation is not uniquely determined** (unlike evaporation in a mixing process, when both θ_L and θ_E and saturation pressure mix conservatively).

Downdraft subsaturation is a result of an draft-scale balance, which can be formulated conceptually in terms of a pressure scale for evaporation Π_E (Betts and SilvaDias, 1979]

$$\Pi_E = \rho g W_d \tau_E \tag{1}$$

where W_d is a characteristic downdraft speed and τ_E a characteristic evaporation time-scale for the water flux of the falling droplets, dependent in turn on their microphysical properties; mean size and number density.

Typical cumulonimbus downdraft outflows have values of Π_E from 30 - 120 mb corresponding to low level relative humidities from 85 - 55%.

[Downdraft sub-saturation related to downdraft RH]

$\Pi_E /p = (1\text{-}RH)/[A+(A\text{-}1)RH]$ where $A = RL/(R_v{*}2C_pT)$

Moist Downdrafts

In many convective parameterizations, these are formulated simply as a *fraction of the updraft mass flux, and often treated as saturated*. This **successfully avoids** the real complexity of the deep convective process by reducing the parametric problem to determining a single updraft mass flux, but it is **an unsatisfactory simplification.**

Unlike shallow convection, deep convection is not a simple mass flux problem, because the precipitation is falling freely and interacting with the atmosphere through evaporation.

For *both* the updraft and downdraft circulations, only one variable θ_E is (approximately) conserved, and **the submodels which handle the microphysics of precipitation fallout and evaporation into downdrafts are critical.**

The widespread use of mass flux models for cumulus parameterization has perhaps partially obscured this important issue.

Extensive discussion in Appendix

The Freezing Level

The freezing level is important in the tropics:

It is in the middle troposphere near 550 mb. Usually it is the level where the profile changes from unstable θ_{ES} to stable. Often there is a visible kink in the thermal θ_{ES} structure, and typically the level of minimum θ_E is near the freezing level. This has been known for many years, but it was not explicitly incorporated as a feature in a convective parameterization scheme until Betts [1986], Betts and Miller [1986].

Undoubtedly the **stratiform precipitation phase change** plays a role in the maintenance of this characteristic structure.

Decay phase of convective mesosystems: inflow peaks at the freezing level
– **two-cell mode** with ascending motion above, and descending motion below
– Seen in the GATE day 245 example

Betts-Miller parameterization scheme

It was clear in the decade after the GATE experiment, one of whose key objectives was to resolve the parameterization and 'scale-interaction' problem, that we had not found a simple solution.

Some advocated more *detailed cloud models* with hierarchies of convective and mesoscale updrafts and downdrafts, and detailed microphysics,

-- but closure: the linking of all the sub-models to each other and to large-scale parameters, was unresolved.

The **Betts-Miller scheme was in response to this**.

– an attempt to formulate the convective forcing in a very simple mathematical way, as lagged convective adjustment towards convective equilibrium profiles of T and q.

– Since we see convection in the tropics adjust the atmosphere towards quasi-equilibrium structures, can we not directly parameterize this process?

– at least as well as trying to get it as an outcome of complex convection sub-models?

Three new concepts:

a) The *moist virtual adiabat* (the reversible adiabat), rather than the pseudoadiabat, as a *reference adiabat* up to the freezing level. This was an inference from observations. The scheme adjusts towards a thermal reference profile, which has a specified instability in the lower troposphere, with respect to this moist virtual adiabat.

b) The *freezing level* was built into the parameterization in calculating the quasi-equilibrium reference profiles, because observationally it appeared to be significant. This was a recognition that the freezing-melting process also plays a role in determining the characteristic thermal structure with a θ_{ES} minimum.

c) *Lagged adjustment* to represent the response time of the convective and mesoscales to changes on the large-scale. This gives a *smoothed convective feedback*, which seems physically more realistic than the on-off behavior of "instantaneous" convection schemes. [and mathematical structure allows the possibility of simplified analytical solutions (Neelin and Yu, 1994), and simple tropical climate models (Seager and Zebiak, 1995,1996).]

Key idea: while convection is occurring, the atmosphere is never allowed to get too far from the type of thermodynamic structures we observe.
– If we constrain a model in this way in the face of large-scale forcing, we are *imposing* the convective sources of heat and moisture, that we would derive by diagnostic methods.

Two decades later this lagged adjustment approach is still being explored
[and we have yet to "solve" the "closure" problem]

Original Betts-Miller Scheme (QJRMS, 1986)

The scheme was designed to adjust the atmospheric temperature and moisture structure back towards a ***reference quasi-equilibrium thermodynamic structure*** in the presence of large-scale radiative and advective processes.

Two distinct reference thermodynamic structures (which are partly specified and partly internally determined) are used for ***shallow and deep convection***.

Formally the convection scheme involves four parts:

- finding cloud-base and cloud-top,
- determining the reference profiles for deep and shallow convection
- the method of distinguishing between deep and shallow convection
- the specification of τ, the adjustment timescale

Formal structure of a lagged adjustment scheme

The large-scale thermodynamic tendency equation can be written in terms of a two-dimensional vector $\boldsymbol{S}(\boldsymbol{\theta}, \boldsymbol{q})$ [or (T^*, p^*)]

$$\partial \overline{\boldsymbol{S}}/\partial \boldsymbol{t} = -\overline{\boldsymbol{V}} \cdot \nabla \overline{\boldsymbol{S}} - \overline{\boldsymbol{\omega}} \partial \overline{\boldsymbol{S}}/\partial \boldsymbol{p} - \boldsymbol{g} \partial \boldsymbol{N}/\partial \boldsymbol{p} - \boldsymbol{g} \partial \boldsymbol{F}/\partial \boldsymbol{p} \qquad (1)$$

where $\boldsymbol{N}, \boldsymbol{F}$ are the net radiative and convective fluxes (including the precipitation flux).

The convective flux divergence is parametrized as

$$-g \partial \boldsymbol{F}/\partial p = (\boldsymbol{R} - \overline{\boldsymbol{S}})/\tau \qquad (2)$$

where

$\boldsymbol{R}$ is the reference quasi-equilibrium thermodynamic structure,

$\boldsymbol{\tau}$ is a relaxation or adjustment time representative of the convective and unresolved mesoscale processes.

Combining (1) and (2) and simplifying the large-scale forcing to the vertical advection, gives

$$\partial\overline{\boldsymbol{S}}/\partial t = -\overline{\omega}\partial\overline{\boldsymbol{S}}/\partial p + (\boldsymbol{R} - \overline{\boldsymbol{S}})/\tau \qquad (3)$$

If the large-scale forcing is steady, on timescales longer than τ, then the atmosphere will reach a quasi-equilibrium with $\partial\overline{\boldsymbol{S}}/\partial t \approx 0$. Then

$$\boldsymbol{R} - \overline{\boldsymbol{S}} \approx \overline{\omega}(\partial\overline{\boldsymbol{S}}/\partial p)\tau \qquad (4)$$

If τ = 1hr, (T-106 global model) $\boldsymbol{R} - \overline{\boldsymbol{S}}$ corresponds to one hour's forcing by the large-scale fields, including radiation.

For deep convection the atmosphere will therefore remain slightly *cooler and moister than the reference state* $\boldsymbol{R}$.

For small τ**,** *the atmosphere will approach the reference state* $\boldsymbol{R}$, so that we may substitute $\boldsymbol{S} \approx \boldsymbol{R}$ in the vertical advection term, giving

$$\boldsymbol{R} - \overline{\boldsymbol{S}} \approx \boldsymbol{\omega\tau}\, \partial \boldsymbol{R} / \partial \boldsymbol{p} \tag{5}$$

from which the convective fluxes can be approximately expressed using (2), as

$$\boldsymbol{F} = \int \frac{(\boldsymbol{R} - \overline{\boldsymbol{S}})}{\tau} \frac{dp}{g} \approx \int \overline{\omega} \frac{\partial \boldsymbol{R}}{\partial p} \frac{dp}{g} \tag{6}$$

Equation (6) shows that the structure of the convective fluxes is *closely linked to the structure of the specified reference profile* $\boldsymbol{R}$.

By adjusting towards an **observationally realistic** thermodynamic structure $\boldsymbol{R}$, we simultaneously constrain the convective fluxes (including precipitation) to have a structure similar to those derived diagnostically from (1), or its simplified form (6), by the budget method (*Yanai et al.*, 1973).

Reference profiles for deep convection

First construct a *first guess thermal profile*, and a *first guess moisture profile*. These are *then corrected to satisfy moist static energy balance*.

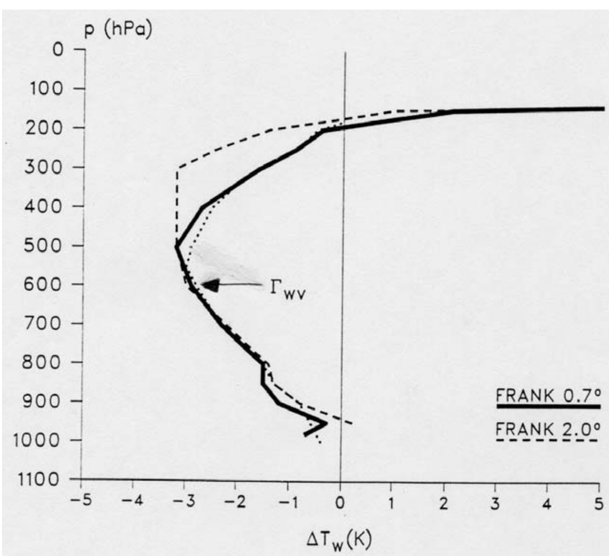
The reference profile for θ is computed up to the freezing level, as a fraction of the slope of the moist pseudo-adiabat.
Defining $\Gamma_w = (\partial\theta/\partial p)_w$ for the moist adiabat, first guess profile is

$$\theta^1{}_R(p) = \overline{\theta}_B + 0.85\ \Gamma_w\ (p_B - p) \qquad (7)$$

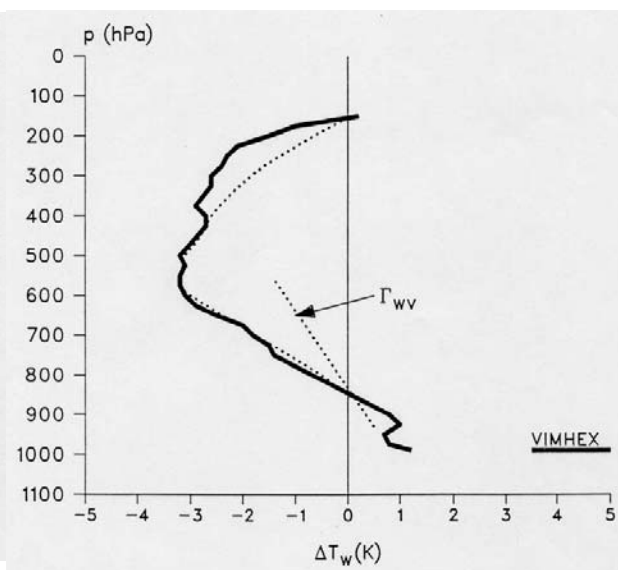
for $p_B < p < p_F$.

A coefficient of 0.9 corresponds to the slope of the wet virtual adiabat: the coefficient of 0.85 is a more unstable profile: a 'compromise' value.

Hurricane core composite



VIMHEX outflow composite



Here we plot ΔT_w, the difference from the pseudoadiabat.

Hurricane core has coefficient of 0.9 (Γ_{wv}); VIMHEX a value of 0.8.

Original scheme we extended this profile down to one level above the surface, and chose a value of $\bar{\theta}_B$ near cloud-base.

Revised 1993 scheme, the deep reference profile near the surface is computed differently from an unsaturated downdraft profile, and $\bar{\theta}_B$ is at a level just above this new boundary layer.

Above the freezing level, the profile returns to the moist pseudo-adiabat at cloud-top. The interpolation is done quadratically in terms of the temperature difference from the wet adiabat, so that

$$T^1{}_R(p) = T_c(p) + [T_R(p_F) - T_c(p_F)](1 - y^2) \quad (8)$$

where $$y = (p_F - p)/(p_F - p_T)$$

These are the curved dotted profiles in the figures above.

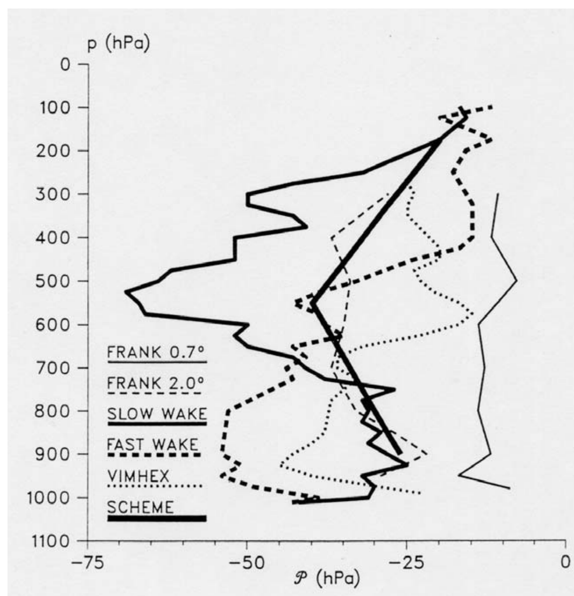
[This involves several small changes from *Betts* and *Miller* (1986), in which the reference profile returned linearly to the environmental temperature at cloud-top, and θ rather than T was used for the interpolation]

Moisture Reference profile q_R

Computed from temperature reference profile by specifying subsaturation $\mathcal{P} = (p^* - p)$ at three levels, cloud-base ($\mathcal{P}_B$), the freezing level ($\mathcal{P}_F$) and cloud-top ($\mathcal{P}_T$) with linear gradients between.

In the present version of the model, the values chosen are ($\mathcal{P}_B$, $\mathcal{P}_M$, $\mathcal{P}_T$) = (-25, -40, -20 mb). Again this is just a compromise to keep the atmosphere from saturating in the presence of forcing.

This figure of mean outflow profiles for different dynamical systems shows the large variability.



Note that

$\mathcal{P}/p = (1-RH)/[A+(A-1)RH]$

where $A = RL/(R_v * 2C_pT)$

so we are simply constraining RH at different levels.

($\mathcal{P}_B$, $\mathcal{P}_M$, $\mathcal{P}_T$) = (-25, -40, -20 mb) correspond to

(RH_B, RH_M, RH_T) ≈ (90, 70, 50%) at p= (950, 550, 200hPa)

However large-scale forcing will move atmosphere towards saturation – from the reference subsaturation

Substituting p and p^* in (5) gives (suffix R for the reference profile)

$$\mathcal{P}_R - \overline{\mathcal{P}} = p_R^* - \overline{p}^* \approx \omega\tau\, dp_R^*/dp \approx \omega\tau \tag{9}$$

since $1 < dp_R^*/dp < 1.1$ for the deep reference profiles which are used. Rearranging gives an approximate value for

$$\overline{\mathcal{P}} \approx \mathcal{P}_R - \omega\tau \tag{10}$$

While the deep convection scheme is operating, the mean vertical advection (if steady for time periods longer than τ) will shift the grid-scale subsaturation $\overline{\mathcal{P}}$ approximately $\omega\tau$ hPa towards saturation from the specified reference state $\mathcal{P}_R$. Thus, although we specify in the present simple scheme a constant global value of the reference structure $\mathcal{P}_R$, $\overline{\mathcal{P}}$ does have a spatial and temporal variability in the presence of deep convection related to that of $\overline{\omega}$.

Note that if we don't want the atmosphere to saturate on the grid-scale in the middle troposphere, then this sets a limit on τ

$$\tau < \mathcal{P}_M/\omega_{max}$$

Correction of reference profiles to satisfy enthalpy constraint

The first guess profiles of $(T_R^1,\ q_R^1)$ are then modified until they satisfy the total enthalpy constraint

$$\int_{p_O}^{p_T} (H_R - \overline{H})dp \ = \ 0 \qquad (11)$$

where $\boldsymbol{H} \ = \ c_p T + \boldsymbol{L}q$ and the integral is through the depth of the convective layer.

The procedure is to calculate

$$\Delta H \ = \ \left(\frac{1}{\Delta p_c}\right)\int_{p_O}^{p_T} (H_R - \overline{H})dp \qquad (12)$$

where Δp_c is the depth of the deep convective layer included in the integral. T_R is then corrected at each level, at constant $\mathcal{P}$, so as to change H_R by ΔH, independent of pressure. This energy correction is iterated once. In *Betts* and *Miller* (1986), this correction was applied at all levels except cloud-top and a shallow surface layer. [In the 1993 scheme the correction is applied at all levels above a model boundary layer, where the adjustment is linked to the downdraft thermodynamics. This involves a modification to (12) to include the two adjustment timescales]

Convective tendencies and precipitation

The convective adjustment, $(R-S)/\tau$, is then applied to the separate temperature and moisture fields as two tendencies (suffix *Cu* for cumulus convection):

$$\left(\frac{\partial T}{\partial t}\right)_{Cu} = \frac{T_R - \overline{T}}{\tau} \qquad (13a)$$

$$\left(\frac{\partial q}{\partial t}\right)_{Cu} = \frac{q_R - \overline{q}}{\tau} \qquad (13b)$$

The precipitation rate is given by

$$\boldsymbol{PR} = \int_{p_O}^{p_T} \left(\frac{q_R - \overline{q}}{\tau}\right)\frac{dp}{g} = -\frac{c_p}{L}\int_{p_O}^{p_T} \left(\frac{T_R - \overline{T}}{\tau}\right)\frac{dp}{g} \qquad (14)$$

No liquid water is stored in the present scheme, and the deep convective adjustment is suppressed if it ever gives $PR < 0$.

[These terms are slightly modified in the 1993 scheme which has a distinct adjustment in the *BL*]

If $PR < 0$, the shallow cloud scheme is called. Since a shallow convective cloud top has not previously been found from a buoyancy criterion, we specified a shallow cloud-top [700hPa].

Different partition between moistening of the atmosphere and precipitation than *Kuo's* scheme.

Given moisture convergence and (say) mean grid-scale upward motion, the model atmosphere moistens with no precipitation until a threshold is reached, qualitatively related to a mean value of $\mathcal{P}_R$, when precipitation starts. However, the model atmosphere continues to moisten (given steady state forcing) until (5) is satisfied, when the moistening ceases, and the 'converged moisture' is then all precipitated.
If the forcing ceases ($\boldsymbol{\omega} \to \mathbf{0}$) then precipitation continues until the atmosphere has dried out to the reference profile again.

– We are essentially controlling (through $\mathcal{P}_R$) the relative humidity of air leaving convective disturbances. In medium range, and particularly climate integrations of a global model, this seems a better way of maintaining the long-term moisture structure of the atmosphere than through constraints on the partition of moisture convergence.
– Clearly the parametrization of the moisture transport or the equilibrium moisture structure is a difficult problem. It depends on the moisture transport from primarily the sub-cloud layer, and the efficiency of the precipitation process, which in turn depends on the cloud and mesoscale dynamics and microphysics.

Choice of adjustment time-scale τ

The role of convective parametrization in a global model is to produce precipitation before grid-scale saturation is reached, both to simulate the real behavior of atmospheric convection, and also to prevent grid-scale instability associated with a saturated conditionally unstable atmosphere.

We can see from (10) that if the convection scheme is to prevent grid-scale saturation $(\overline{\mathcal{P}} = 0)$, there is a constraint on τ,

$$\tau < \mathcal{P}_R / \omega_{\max} \qquad (15)$$

where $\omega_{\max}$ is a typical maximum ω in say a major tropical disturbance. With $\mathcal{P}_R \sim -40$ *mb* in the middle troposphere, we have found this suggests an upper limit on τ, which is $\sim$ two hours for the ECMWF T-63 spectral model and $\sim$ one hour for the T-106 model, with smaller values at higher resolutions. We recommend that τ should be set so that the model atmosphere nearly saturates on the grid-scale in major convective disturbances. In a numerical model a lagged adjustment, rather than a sudden adjustment at a single time-step, has the advantage of smoothness, with less of a tendency to 'blink' on and off at grid-points in a physically unrealistic way.

Betts (1997) suggested relating τ to gravity wave propagating speeds .. gives similar values

TABLE 1. Adjustment times as a function of horizontal scale for two primary wave-modes.

	Wave-speed	Adjustment time τ		
	C(ms^{-1})	(60 km)	(120 km)	(400 km)
Wave-Mode 1 (Deep Troposphere)	50	20 min	40 min	2.2 hrs
Wave-Mode 2 (Inflow at Freezing Level)	25	40 min	80 min	4.4 hrs

Weaknesses of scheme

– *adjustments at top and bottom of profile* sensitive to first guess, and the energy correction to reference profiles

– particularly important for the BL

.. this lead to addition of downdraft

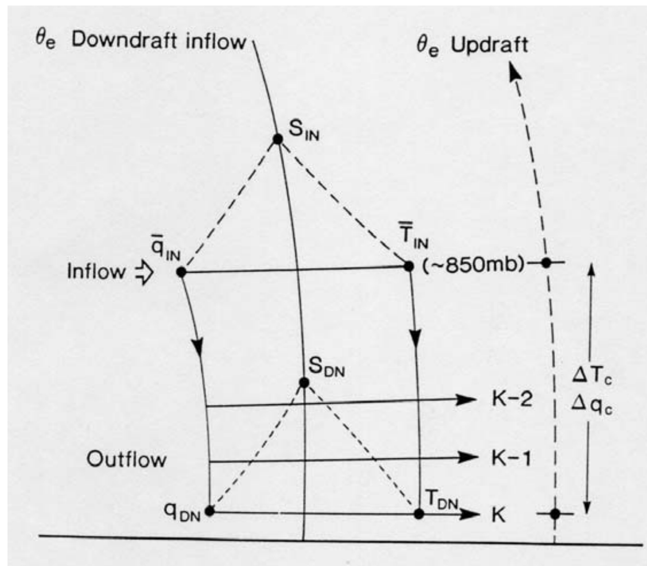
– *Vapor adjustment physics less justifiable than thermal adjustment*

– *Different dynamical/microphysical regimes not represented*, nor understood in parametric terms

Addition of a low-level Downdraft (1993)

We introduce an unsaturated downdraft reference profile, which starts at a downdraft inflow level with the mean properties at that level, and descends at constant θ_E and constant subsaturation: that is, the temperature and moisture paths are parallel to a moist adiabat. In the 1993 code, the downdraft originates at a single level near 850 mb.

Schematic



The reference profile, T_R, q_R , for the three lowest model levels (K to K-2) are set equal to the downdraft outflow properties.

$$T_R = T_{DN} = \overline{T}_{IN} + \Delta T_c \quad (16a)$$

$$q_R = q_{DN} = \overline{q}_{IN} + \Delta q_c \quad (16b)$$

where $\overline{T}_{IN}$, $\overline{q}_{IN}$ are the grid mean values at the downdraft inflow level, and ΔT_c, Δq_c are the changes of T,q along the downdraft descent path (defined positive).

This *simple downdraft* parametrization has other advantages, besides being computationally efficient. It couples the relative humidity in the BL to the constrained subsaturation $\mathcal{P}$ at higher levels and gives tendencies towards a subsaturated moist adiabatic structure (see equation (18)). In the BL , the tendencies due to cumulus convection are

$$\left(\frac{\partial T}{\partial t}\right)_{Cu} = \frac{T_R - \overline{T}}{\tau_{BL}} = \frac{\Delta T_c - \Delta \overline{T}}{\tau_{BL}} \tag{17a}$$

$$\left(\frac{\partial q}{\partial t}\right)_{Cu} = \frac{q_R - \overline{q}}{\tau_{BL}} = \frac{\Delta q_c - \Delta \overline{q}}{\tau_{BL}} \tag{17b}$$

where τ_{BL} is the adjustment time of the BL, discussed in the next section; and $\Delta\overline{T}$, $\Delta\overline{q}$ are the vertical differences in the mean structure between downdraft inflow and BL outflow levels. The BL tendencies become zero if

$$\begin{aligned} \Delta\overline{T} &= \Delta T_c \\ \Delta\overline{q} &= \Delta q_c \end{aligned} \tag{18}$$

that is, if the mean profiles become parallel to the moist adiabat. (The downdraft will not be saturated however unless the downdraft inflow is saturated). Typically, unless (18) is achieved, $\Delta\overline{T} > \Delta T_c$, and $\Delta\overline{q} > \Delta q_c$ and the downdraft cools and dries the BL.

BL adjustment time, τ_{BL}

This is computed by coupling the evaporation into the downdraft to the precipitation rate (*PR*). If we define

$$EVAP = \int_{P_O}^{P_{BL}} (\Delta.V)_D \ \Delta q_c \ dp/g = \alpha \ PR \tag{19}$$

where we assume constant divergence $(\Delta.V)_D$ of the downdraft in the BLα, and a constant of proportionality . The BL timescale τ_{BL} is given by

$$1/\tau_{BL} = (\Delta.V)_D = \alpha \ PR/\int_{P_O}^{P_{BL}} \Delta q_c \ dp/g \tag{20}$$

This couples the BL timescale to the precipitation driving downdraft processes. We set $\alpha = -0.25$ globally to represent a precipitation efficiency of order 0.8, consistent with tropical budget studies The parameter could be made a function of windshear (e.g. *Fritsch* and *Chappell*, 1980). Typically τ_{BL} is longer than τ, so that the boundary layer adjustment is slower and smoother than in the original version of the scheme, as well as being well-defined in terms of a physical process. We have found much smoother precipitation patterns in this revised version of the scheme, presumably because the convection scheme is less apt to be shut off by rapid changes of θ_E in the BL.

[Modification to energy correction .. see 1993 text]

Other issues

Should we parameterize the mesoscale?

The Key Convective Modes

Arakawa and Chen [1987].... used canonical correlation analyses on the GATE Phase III data [of Esbensen and Ooyama 1983] and an Asian data set [from He et al. 1987] to show there were **three principal modes of coupling of (Q_1-Q_R) and Q_2.**

Mode 1 **is the principal deep convection mode** associated with net precipitation and a **single cell of mean upward vertical motion in the troposphere**, although within that there are moist updrafts and downdrafts.

$$LF_{PR}(0) = \int_0^{z_T} (Q_1 - Q_R)\,dz = \int_0^{z_T} Q_2\,dz \tag{3}$$

There is a **net upward flux of θ_E**, and moist static energy h, peaking in the mid-troposphere where the (Q_1-Q_R) and Q_2 curves cross.

$$(C_p T/\theta_E) F_{\theta_E}(z) \approx F_h(z) = -\int_0^{z_T} (Q_1 - Q_2 - Q_R)\,dz \tag{4}$$

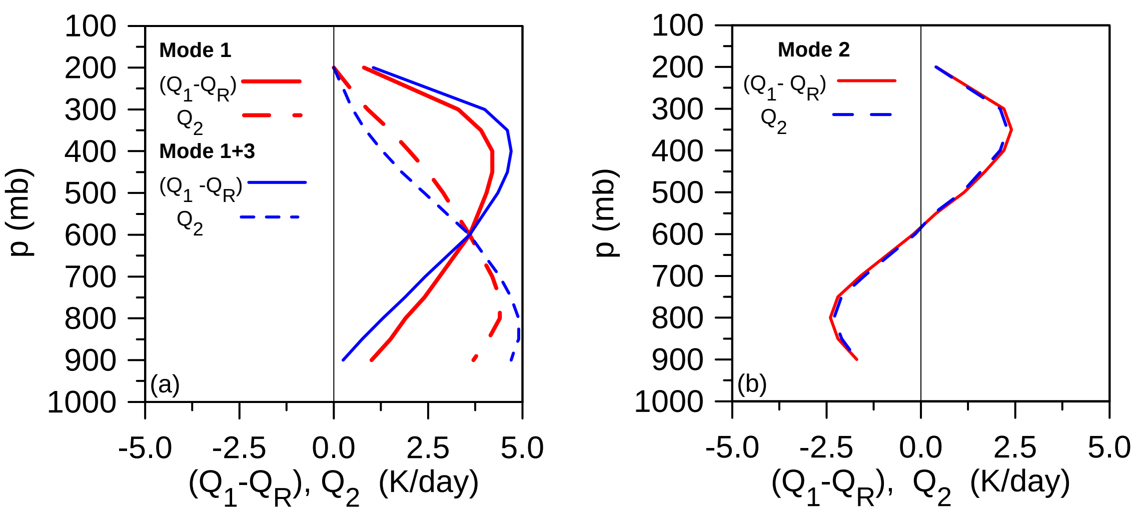
Mode 3... is a modulation of Mode 1, which *increases* the mid-tropospheric θ_E flux, without impact on net precipitation.
Upward θ_E flux is *not uniquely* coupled to the precipitation.

Precipitation...... heating and a deep tropospheric ascent mode.
Upward θ_E flux..... lowers boundary layer θ_E

Mode 2 is described by Arakawa and Chen as the component representing deviations of "large-scale" condensation and evaporation, with $(Q_1 - Q_R) = -Q_2$

Heating over cooling couplet with no net precipitation
– zero θ_E flux

This diagnostically derived mode can be thought of as the signal coming from the ***variable presence of mesoscale anvils*.**
The key consequence of this is to force a **2-cell** vertical structure with ascent over descent, and a larger-scale convergence in mid-levels near the freezing level.



Minimum requirements for a convective parameterization scheme

is whether it can represent these **3 modes** correctly.

a) Deep convective precipitating mode with an upward θ_E flux, not uniquely coupled to the precipitation (Modes 1 and 3)

b) Heating/cooling couplet with no net precipitation and no θ_E flux (Mode 2).

A scheme also **needs sufficient closures** to be able to determine the magnitude of the net precipitation, θ_E flux and the heating over cooling couplet; and perhaps their time evolution for an evolving unresolved mesoscale convective system.

Does the couplet Mode 2 have to be parameterized at all?

If it is "large-scale" precipitation, why can't the grid-scale processes handle it (provided there is an adequate prognostic cloud-scheme being fed liquid and solid precipitation from the convective scale)?

We are approaching the heart of the so-called
"scale-interaction problem."
What scales are well represented by the large-scale model?

It is clearly unreasonable to expect a hydrostatic global climate model with a horizontal grid of 250 km to represent the mesoscale at all, but can a hydrostatic model with a 50 km grid develop a crude representation of a mesoscale anvil?

The **key test** I would propose is **whether the mid-level convergence develops** in tropical convective systems in the model at the end of their lifecycle. If not, I would argue **it should be forced** by parametrically representing the Mode 2 couplet.

In nature all the scales interact dynamically and can evolve together

– but the convective and mesoscales have shorter time scales than the 'large-scale'.

– Because we only simulate the dynamics of the large-scale in our global models, the **faster processes must be parameterized.**

Just as a convection scheme, by introducing precipitation before saturation on the grid-scale is reached, can change the phasing of large-scale dynamical development, so if we introduce a parameterized mesoscale couplet forcing, this too will feed back on the large-scale model dynamics sooner, than if we wait for grid-scale processes to reach saturation.

Since we know that this inflow at the freezing level is dynamically important in the tropics, it is likely that the impact of this Mode 2 parameterization will be significant.

Conclusions

After 3 decades of convective parameterization
– no consensus on the "best" scheme.
– small changes in formulation have large impacts in tropics
[and may have in mid-latitudes – see "diurnal talk"]

Should the mesoscale be parameterized?

Will nested high resolution models "solve" problem
[or simply shift it to the microphysical/diffusive grid-scale parameterizations]

Do we have the radiation coupling correct on all scales?
[yesterday's talk]

Appendix: Mass Flux Representation of Deep Convective Updrafts and Downdrafts [from *Betts, 1997*]

Condensed summary to illustrate the key issues. The subgrid-scale heating and drying by convective updrafts and downdrafts can be written in bulk form as *[new equation #'s]*

$$(Q_1 - Q_R) = -\frac{\partial F_{PR}}{\partial z} - \frac{\partial}{\partial z}[M_u(s_{Lu} - \bar{s})] + \frac{\partial}{\partial z}[M_d(s_d - \bar{s})] \tag{5}$$

$$-Q_2 = \frac{\partial F_{PR}}{\partial z} - \frac{\partial}{\partial z}[M_u(q_{Tu} - \bar{q})] + \frac{\partial}{\partial z}[M_d(q_d - \bar{q})] \tag{6}$$

The updraft and downdraft mass fluxes are M_u, M_d respectively, and they satisfy mass conservation equations

$$\partial M_u / \partial z = \varepsilon_u - \delta_u \tag{7}$$

$$-\partial M_d / \partial z = \varepsilon_d - \delta_d \tag{8}$$

where ε, δ represent entrainment and detrainment rates. The bulk properties of the updraft are its liquid water static energy s_{Lu} and total water q_{Tu}; the downdraft is assumed to have no cloud water, so its properties are s_d, q_d. The environmental mean with properties $\bar{s}$, $\bar{q}$ is also assumed unsaturated.

***F*$_{PR}$(z) is the flux of precipitation**, which is related to 3 terms

$$-\partial F_{PR} / \partial z = C_u - E_d + Q_M \tag{9}$$

where C_u is the fallout of precipitation from the updraft and E_d is the evaporation of falling precipitation into the downdraft.

Following Arakawa and Chen [1987] and Cheng and Yanai [1989], in addition to the convective terms, we include **a mesoscale condensation/evaporation couplet term, Q_M, which is not linked to a mass circulation, and which has both zero moist static energy (*h*) flux and no net precipitation flux.** Thus Q_M satisfies

$$Q_M = Q_{1M} = Q_{2M} \tag{10a}$$

and

$$\int_0^{z_T} Q_M \, dz = 0 \tag{10b}$$

From **energy conservation in the updraft**, which entrains and detrains and condenses precipitation as it ascends, one can write the updraft budget equation

$$\frac{\partial}{\partial z}(M_u s_{Lu}) = C_u + \varepsilon_u \bar{s} - \delta_u s_{Lu} \tag{11}$$

Similarly for the **downdraft**

$$-\frac{\partial}{\partial z}(M_d s_d) = -E_d + \varepsilon_d \bar{s} - \delta_d s_d \tag{12}$$

where in both (11) and (12) it is assumed that air entrained into both updraft and downdraft has the properties of the mean environment. Substituting (11) and (12) in (5) and (6), using (7), (8), and (9), and rearranging, gives

$$Q_1 - Q_R = M_c \, \partial \bar{s} / \partial z + \delta_u (s_{Lu} - \bar{s}) + \delta_d (s_d - \bar{s}) + Q_M \tag{13}$$

$$-Q_2 = M_c \partial \bar{q} / \partial z + \delta_u (q_{Tu} - \bar{q}) + \delta_d (q_d - \bar{q}) - Q_M \tag{14}$$

where the net convective mass flux is (defining both M_u and M_d as positive)

$$M_c = M_u - M_d \tag{15}$$

...The heart of the parametric mass flux representation

Cho [1977] recognized that if the convective outflows are considered to be at **buoyancy equilibrium** after evaporating any remaining cloud water (note this needs a precip. parameterization!), then one can formally drop the detrainment terms in (13) (if virtual temperature effects are neglected), and calculate an M_c from (also dropping the mesoscale term)

$$Q_1 - Q_R = M_c \partial\bar{s} / \partial z \tag{16}$$

It could perhaps be argued that this is a satisfactory treatment, except near the surface, where cold downdraft outflows cannot sink to buoyancy equilibrium.
Cho noted that even if M_c is calculated from (16), there is no equivalent condition to buoyancy equilibrium in the moisture budget.

The **moisture content of convective outflows** must be determined in (14) to solve the parameterization problem, since it cannot be assumed that $q_{Tu} = q_d = \bar{q}$.
Downdraft outflows are typically unsaturated and must be modeled.

The precipitation fallout determines the water content of updraft outflows and they too will not be saturated, after sinking to buoyancy equilibrium [Betts 1982a].

Net vertical mass transport of the deep convective mode is directly related to the net precipitation. Integrating (16) gives

$$\int_0^{z_T} (Q_1 - Q_R)\,dz = \int_0^{z_T} (M_c \partial \overline{s} / \partial z)\,dz \tag{17}$$

Substituting (9) in (5) and integrating through the troposphere gives

$$\int_0^{z_T} (Q_1 - Q_R)\,dz = F_{PR}(0) = \int_0^{z_T} (C_u - E_d)\,dz \tag{18}$$

since the convective fluxes disappear at the integration limits and the mesoscale term disappears using (10b). (If we had not dropped the surface sensible heat flux it would also be included here).

Adding (13) and (14), **the mesoscale term**, which has no h transport, disappears and we get

$$Q_1 - Q_2 - Q_R = M_c \frac{\partial \overline{h}}{\partial z} + \delta_u h_u' + \delta_d h_d' \tag{19}$$

where $h_u' = h_u - \overline{h}$, and $h_d' = h_d - \overline{h}$.

In **Betts [JAS 1973]**, the inflows and outflows were measured directly and the terms in (19), including separate updraft and downdraft mass fluxes were evaluated.

In other diagnostic studies using sonde networks to derive $Q_1 - Q_2$ (Q_R is usually calculated), (19) cannot be inverted by itself as it contains 2 unknowns, the updraft and downdraft mass fluxes. **Only if downdrafts are neglected**, can (19) be immediately inverted, given a cloud model for h_u to derive a net convective mass flux M_c **[Yanai et al. 1973]**.

The ratio of downdraft to updraft mass fluxes can be prescribed to solve (19). [**Johnson, 1976]**

Any two of (13), (14) and (19) can however be regarded as independent.
Nitta (1977) showed that, **if a precipitation parameterization is introduced** to determine the remaining cloud water in s_{Lu}' in (13), and **the downdrafts are assumed to be saturated,** then (13) and (19) can be solved simultaneously for both an updraft and downdraft mass flux.

The key conclusion is that the mass flux representation of convection is only adequate to the extent that the detrainment terms in (13), (14) and (19) can be either neglected or calculated.

A mass flux parameterization for deep convection must calculate both an updraft and a downdraft mass circulation and the properties of the outflows of both the updrafts and downdrafts. All these depend on microphysical and cloud-scale dynamical processes: a satisfactory general solution has not yet been found.

Convective precipitation is linked to a convective mass flux (see (17 and (18)) but **this alone does not determine the upward θ_E**

flux, which is also linked to the environmental θ_E structure, and the convective-scale processes.

Combining (4) and (19), consider the integral to the freezing level z_F in the mid-troposphere

$$(C_pT/\theta_E)F_{\theta_E}(z_F) = F_h(z_F) = -\int_0^{z_F} M_c \frac{\partial \overline{h}}{\partial z}\,dz - \int_0^{z_F} \delta_u h_u{}'dz - \int_0^{z_F} \delta_d h_d{}'dz \qquad (21)$$

+ (Neglect) +

At the freezing level, $\overline{h}$ is typically a minimum, so below z_F, $\partial \overline{h}/\partial z$ is negative, and the first term is positive. The downdraft outflows have typically $h_d{}'$ negative, so the third term is also positive. If we neglect updraft outflows in the lower troposphere (which means ignoring shallow clouds), it is clear that the strength of mid-tropospheric upward deep convective flux of h (and θ_E) is related to the net convective mass flux, the value of the $\overline{h}$ minimum, and the strength and properties of the downdrafts. Since downdrafts bring down mid-tropospheric low $\overline{h}$ air,
a low value of mid-tropospheric $\overline{h}$ contributes in both terms to a larger upward *h* flux.

It is this upward flux of h or θ_E which plays a key role in the convective interaction with the surface fluxes as discussed earlier (see also Raymond [1996]).