A Lagged Mixing Parameterization for the Dry Convective Boundary Layer

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ABSTRACT

A lagged mixing parameterization is proposed for the dry convective boundary layer in which the entrainment rate is controlled by the adjustment time chosen. Data from a growing boundary layer over land are used as illustration.

1. Introduction

In the atmosphere dry convective turbulence transports heat and moisture away from the surface and mixes down (or entrains) warmer and typically drier air from the boundary layer (BL) top. The prediction of the time evolution of the BL thermodynamics depends on accurately predicting both the surface fluxes and the entrainment fluxes at BL top. Simple mixed-layer models have been used to describe this process for many years (Betts 1973; Carson 1973; Tennekes 1973) by using a closure equation to couple the vertical heat flux at BL top to the surface heat flux:

$$F_{I\theta_v} = -A_R F_{0\theta_v}, \tag{1}$$

where $F_{I\theta_v}$, $F_{0\theta_v}$ are, respectively, the virtual heat fluxes (here F will represent a flux in watts per square meter) at the inversion and the surface, and A_R is a closure parameter. The value of A_R is not well known. Modeling and laboratory studies have suggested $A_R \sim 0.2$ for free convection (see Stull 1988), but some recent field experiments have estimated larger values of $A_R \sim 0.4$ (Betts 1992; Betts et al. 1992; Culf 1992). Many simple parameterizations have been used in numerical models to represent the dry BL; from dry-adiabatic adjustment to mixed-layer models (Arakawa and Schubert 1974) to downgradient diffusion models (Louis 1979) to higher-order closure models (Janjić 1990). Some models, such as downgradient diffusion models (Louis 1979), will not produce the entrainment flux at the stable inversion. Ad hoc fixes have been used in global models to ensure the entrainment fluxes are represented (Beljaars and Betts 1993), since without them the diurnal thermodynamic cycle over land is poor (Betts et al. 1993).

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One purpose of this note is to show how a simple lagged mixing model can describe the evolution of the dry BL, where the choice of adjustment or mixing timescale controls the entrainment rate. The analysis, which uses a parcel model to determine BL depth, also illustrates the difficulty of properly coupling sequential parameterizations for physical processes. Here we use the dry mixed layer for illustration, but a similar issue arises for the parameterization of the shallow cumulus BL.

2. Simple lagged mixing model

Betts (1986) and Betts and Miller (1986) introduced a lagged mixing parameterization for shallow cumulus and cumulonimbus. They formulated the convective transports in terms of adjustment toward characteristic reference profiles for potential temperature θ and moisture q, with an adjustment time τ. For the nearly well-mixed dry BL, an especially simple form of this parameterization (which to my knowledge has not been explored) is to express the convective term as

$$\left(\frac{\partial \bar{S}}{\partial t}\right)_{\text{conv}} = \frac{\hat{S} - \bar{S}}{\tau}. \tag{2}$$

This parameterization mixes the horizontal mean thermodynamic properties $\bar{S}$ toward a vertically homogeneous state $\hat{S}$, the layer mean, with a timescale τ. This parameterization now reduces to (a) defining the convective layer and (b) defining the timescale of mixing.

Entrainment is then determined by the choice of BL top (defined here by the parcel method) and mixing timescale τ. We neglect radiative processes within the boundary layer.

3. Illustrative example

The growth of a dry mixed layer will be used as illustration. The initial condition will be a mean First

ISLSCP (International Land Surface Climatology Project) Field Experiment (FIFE) sounding at 1700 UTC (approximately 1100 LST) in July 1987 near Manhattan, Kansas (from Betts and Ball 1994). We will consider BL evolution for the time interval 1700–1830 UTC for which we have a pair of mean sonde profiles. The mean surface sensible and latent heat fluxes for this time period are 110 and 383 W m^{-2}, giving a surface virtual heat flux of 137 W m^{-2}. We use for convenience a time step Δt of 540 s (0.1 of the time interval of 90 min between the mean sondes) and add the surface fluxes every time step as equal increments to a layer just above the surface, which is 8.7 mb thick. Suppose this first layer then has potential temperature θ_{v0} after this addition of heat. The dry BL is then defined here as the layer for which $\theta_v < \theta_{v0}$ (i.e., θ_{v0} is used to define a BL top as the equilibrium level of parcel lifted dry adiabatically from the surface). Parameterization (2) is then used to distribute the surface fluxes through this BL. Figures 1 and 2 compare the observed θ_v and q at 1830 UTC soundings with two model integrations from the same observed 1700 UTC sounding using two adjustment times of 1000 and 1400 s. The parameterization does a good job simulating the mixed layer and its evolution, and simulating BL-top entrainment since parcels heated at the surface rise to their equilibrium level. The effective entrainment is too small for $\tau = 1000$, but about right for the $\tau = 1400$ s. For $\tau > 1400$, entrainment is greater than observed (not shown).

4. Analytic solution

This model is sufficiently simple that we can explore it analytically. BL-top entrainment depends on θ_{v0}.

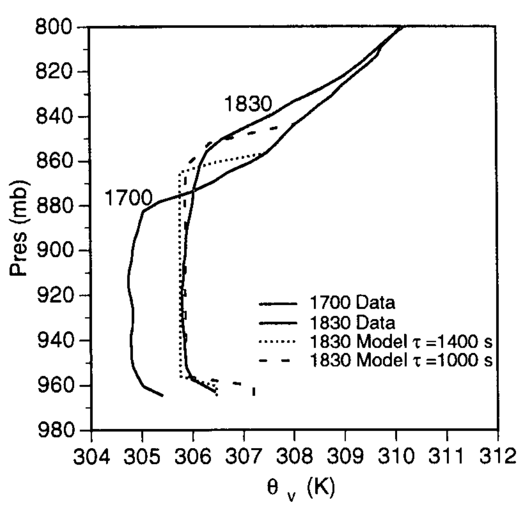

FIG. 1. Comparison of FIFE July average sonde θ profiles at 1700 and 1830 UTC (solid), with integration of mixing parameterization from the 1700 UTC observed profile to 1830 UTC with an adjustment time of 1000 (dotted) and 1400 s (dashed).

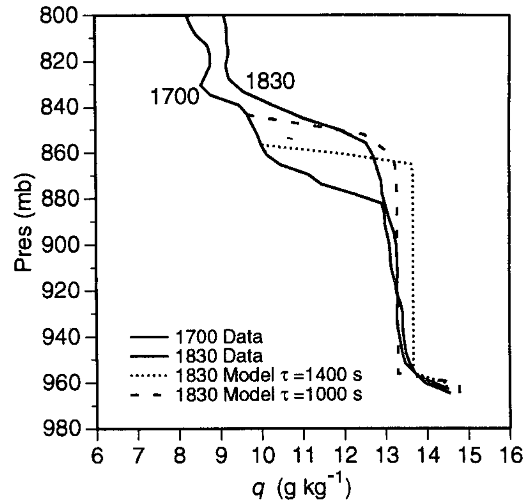

FIG. 2. As in Fig. 1 but for q profiles.

Here θ_{v0} is determined within a few time steps by a balance of the added surface energy and the lagged adjustment. This balance can be written

$$\frac{(\hat{\theta}_v - \theta_{v0})}{\tau}\Delta t + \frac{gF_{0\theta_v}\Delta t}{C_p\Delta p_s} = 0, \tag{3}$$

where $F_{0\theta_v}$ is the surface virtual heat flux, and Δp_s is the thickness (defined positive) of the "surface layer" to which the surface fluxes are added. The time step Δt cancels, giving the θ_v excess of the first layer as a lagged value related to adjustment time, surface heat flux, and surface layer depth

$$\Delta\theta_v = \theta_{v0} - \hat{\theta}_v = \tau\left(\frac{gF_{0\theta_v}}{C_p\Delta p_s}\right). \tag{4}$$

This involves the velocity scale $\omega_s = \Delta p_s/\tau$. For $\tau = 1400$ s, (4) gives $\Delta\theta_v = 2.16$ K. The longer τ (or smaller Δp_s), the larger $\Delta\theta_v$, the higher the parcel equilibrium level, and correspondingly the higher the model entrainment. The inversion strength produced by this parametric model is $\Delta\theta_{vI} = \Delta\theta_v$ since the surface θ_{v0} determines θ_v at BL top.

It could be argued that this is a very oversimplified model for the dry BL where it is better to compute the surface and BL fluxes simultaneously. However, the simplicity will yield illuminating solutions. Moreover, consider a cumulus parameterization where the code for the dry convective BL model is first called, followed by a cumulus parameterization using updated sub-cloud layer parameters. A similar interaction of sequential components of the model parameterization may determine the entrainment rate for the cumulus BL.

In this simple model, the entrainment rate ω_e can be extracted explicitly, with some simplifying assumptions. Write the mixed-layer budget as

$$\frac{\partial \hat{\theta}_v}{\partial t} = \frac{g(F_{0\theta_v} - F_{I\theta_v})}{C_p \Delta p}, \tag{5}$$

where Δp is the BL depth (in p coordinates, defined positive), and $F_{I\theta_v}$ is the inversion level virtual heat flux (the entrainment flux). If we assume the inversion strength is constant, then

$$\frac{\partial \hat{\theta}_v}{\partial t} = \frac{\partial \theta_v^+}{\partial t} = \Gamma_v^+ \omega_e, \tag{6}$$

where θ_v^+ is just above the inversion, and Γ_v^+ is the gradient of θ_v above the BL, also defined positive. The entrainment flux can be written as

$$gF_{I\theta} = -\omega_e C_p \Delta\theta_v. \tag{7}$$

Combining (4), (5), and (6) gives the entrainment rate

$$\omega_e = \left(\frac{C_p \Gamma_v^+ \Delta p}{g F_{0\theta_v}} - \frac{\tau}{\Delta p_s} \right)^{-1}. \tag{8}$$

The first term is the reciprocal of the BL growth by encroachment, related only to the surface θ_v flux

$$\omega_{\rm enc} = \frac{g F_{0\theta_v}}{C_p \Gamma_v^+ \Delta p}. \tag{9}$$

Thus, ω_e is determined by two terms

$$\omega_e = \left(\frac{1}{\omega_{\rm enc}} - \frac{\tau}{\Delta p_s} \right)^{-1} = \frac{\omega_{\rm enc}\omega_s}{\omega_s - \omega_{\rm enc}}, \tag{10}$$

where $\omega_s = \Delta p_s/\tau$. Note that $\omega_e \to \omega_{\rm enc}$ as $\tau \to 0$, and the entrainment rate becomes large as $\tau \to \Delta p_s/\omega_{\rm enc}$. In this parameterization, entrainment is related to the balance of two processes: how fast the surface flux warms the mixed layer and how large a temperature excess is maintained in the near-surface superadiabatic layer by the surface heating before heat is transferred up into the mixed layer. Our parameterization of the latter is very simple. We added all the heat to one layer of thickness Δp_s, so that this term looks like an artifact. Clearly, however, one could relate Δp_s to the depth of the surface layer and τ to a dry convective timescale. Our value of $\Delta p_s = 8.66$ mb corresponds to approximately 100 m. Our value of $\tau = 1400$ s (23 mins) is a realistic timescale for mixing in a dry BL. In general, however, τ must be chosen to give a realistic entrainment rate.

The ratio of the entrainment flux to the surface flux given by (1) is also related to these same velocity scales

$$A_R = \frac{F_{I\theta v}}{F_{0\theta v}} = \frac{C_p \omega_e \Delta\theta_v}{g F_{0\theta}} = \frac{\omega_e \tau}{\Delta p_s} = \frac{\omega_{\rm enc}}{\omega_s - \omega_{\rm enc}}. \tag{11}$$

Clearly τ can be chosen to give a specific value of A_R with this model. For the example in Figs. 1 and 2, τ was chosen to approximately match the observed evolution. For these data, the velocity scales are $\omega_{\rm enc} = 0.21$ Pa s^{-1} (an average for the 90-min time step for which $\Delta p = 99.7$ mb) and $\Delta p_s/\tau = 0.62$ Pa s^{-1}, giving $\omega_e = 0.32$ Pa s^{-1} from (10) and $A_R = 0.5$ from (11). This value of A_R fits well the value of 0.45 derived from the data for these 90-min time step and it is characteristic of the FIFE data (Betts 1992; Betts and Ball 1994). (This time step was chosen for illustration because of its representivity: in general one cannot derive accurate budgets from averaged sonde differences from a single time step.) The corresponding estimates of the inversion level fluxes are

$$F_{I\theta_v} = -C_p\left(\frac{\omega_e}{g}\right)\Delta\theta_{vI}$$
$$= (1006)(0.32/98)2.18 = -72 \text{ W m}^{-2},$$

$$F_{Iq} = -L\left(\frac{\omega_e}{g}\right)\Delta q_I$$
$$= (2501)(0.32/9.8)3.6 = 294 \text{ W m}^{-2}.$$

These are comparable to the values derived from the 1700, 1830 sonde pair (Betts and Ball 1994) by the budget method (-94, 435 W m^{-2}) and the inversion rise method (-82, 326 W m^{-2}).

5. Conclusions

This lagged mixing parameterization can be an adequate simple model for the dry BL evolution. The adjustment timescale in the convective mixing scheme can be chosen to give any BL entrainment rate. Unfortunately, BL entrainment rates are not well known. This simple model also shows that the sequential use of parameterizations needs careful study. It may be more important to understand the sequential interaction of different parameterizations in a large-scale model (such as the surface, dry BL, cloudy BL, and radiation schemes) rather than to simply add greater detail to any one scheme.

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