

# Boundary layer equilibrium

## – oceans and land

[2004]

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Based on:

Betts, A.K., 1997: Trade Cumulus: Observations and Modeling. Chapter 4 (pp 99-126) in “*The Physics and Parameterization of Moist Atmospheric Convection*, Ed. R. K. Smith, NATO ASI Series C: Vol. **505**, Kluwer Academic Publishers, Dordrecht, 498pp.

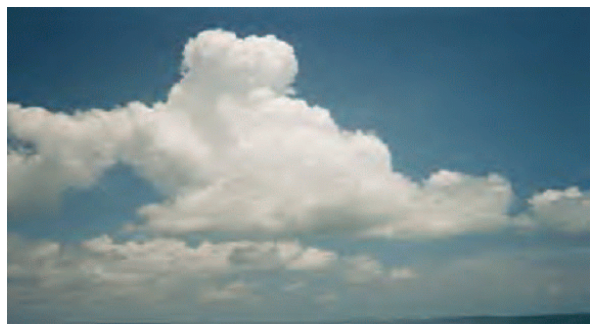
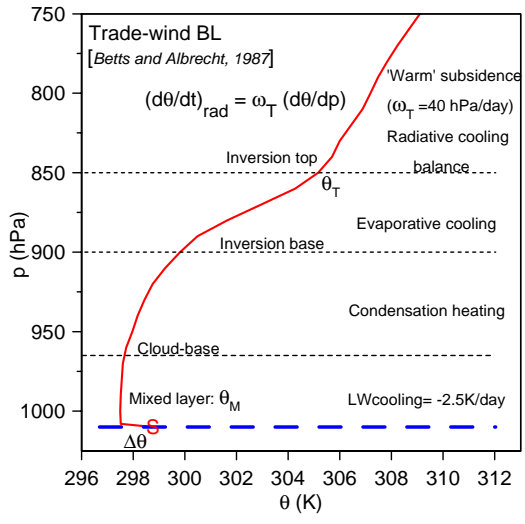
Betts, A. K., and B. A. Albrecht, 1987: Conserved variable analysis of boundary layer thermodynamic structure over the tropical oceans. *J. Atmos. Sci.*, **44**, 83-99.

Betts, A. K. and W. Ridgway, 1988 : Coupling of the radiative, convective and surface fluxes over the equatorial Pacific. *J. Atmos. Sci.*, **45**, 522-536.

Betts, A.K. and W. L. Ridgway, 1989: Climatic equilibrium of the atmospheric convective boundary layer over a tropical ocean. *J. Atmos. Sci.*, **46**, 2621-2641.

Betts, A. K., 2000: Idealized model for equilibrium boundary layer over land. *J. Hydrometeorol.*, **1**, 507-523.

Betts, A. K., B. Helliker and J. Berry, 2004, Coupling between CO<sub>2</sub>, water vapor, temperature and radon and their fluxes in an idealized equilibrium boundary layer over land. *J. Geophys. Res.* (In revision).



Why is mixed layer cooler than the ocean SST?

LW cooling = -2.5 K/day

Clouds redistribute heat and water and modify radiative balance

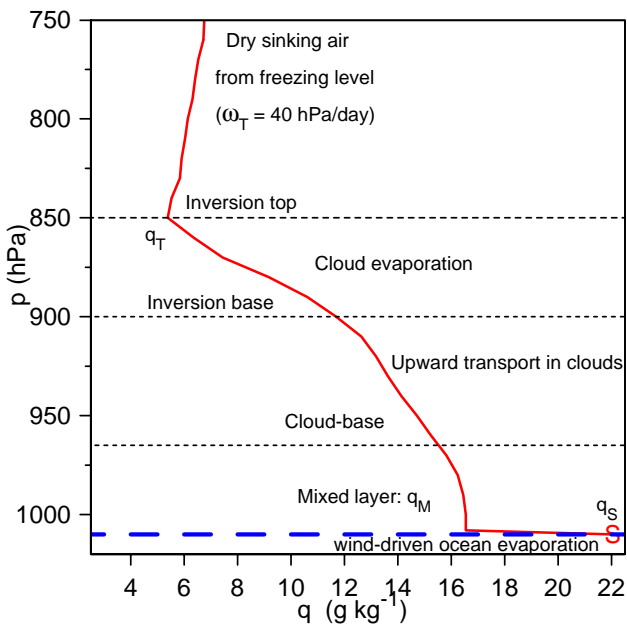
Equilibrium for whole layer:

$$0 = (g/C_p) \Delta R_{\text{net}} + \omega_0 \Delta\theta + \omega_T (\theta_T - \theta_M)$$

-40	+10	+30 W m <sup>-2</sup>
cooling	surface flux	subsidence

Surface velocity scale:  $\omega_0 = \rho V_0 C_D \approx 90 \text{ hPa/day}$

Subsidence:  $\omega_T \approx 40 \text{ hPa/day}$



Why is the mixed layer not saturated, as the air blows over ocean?  
Evaporation from ocean is balanced by subsidence of dry air above.

$$0 = \omega_0 (q_S(\text{SST}) - q_M) + \omega_T (q_T - q_M)$$

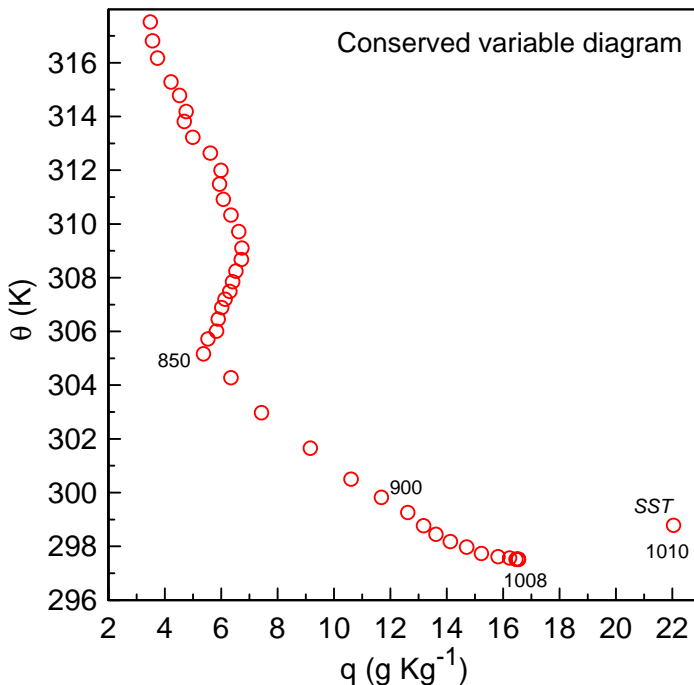
$$q_M = [\omega_0 q_S(\text{SST}) + \omega_T q_T] / (\omega_0 + \omega_T)$$

A weighted average

$$q_M = [90 \cdot 22 + 40 \cdot 5] / 130 = 16.7 \text{ g/kg}$$

$$\text{so } \theta_{EM} \approx 346\text{K}$$

$$\text{cloud-base} \approx 960\text{hPa}$$



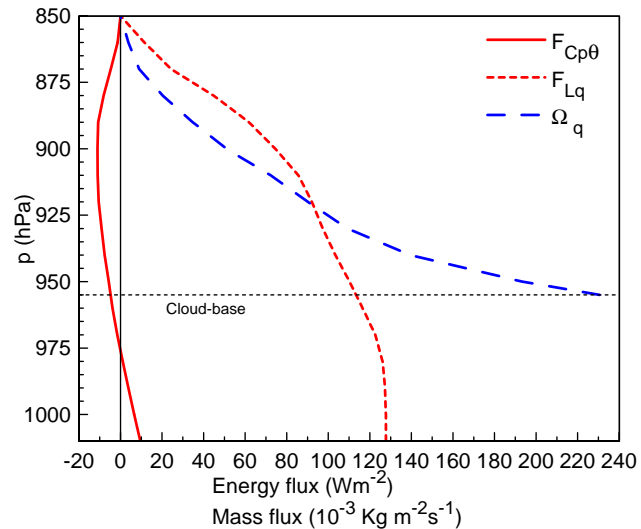
Can think of the two balances on a ‘conserved parameter’ diagram:  
“Mixing” of surface point and 850hPa point, modified by radiation.

## Relate equilibrium structure to convective fluxes: $F_q$ , $F_\theta$ [illustration]

Assume  $\omega = 40\text{hPa/day}$  in cloud layer, below cloud-base decreases linearly to zero at surface.

Assume radiative cooling

$$\partial\theta_{\text{Rad}}/\partial t = -2.4 \text{ K/day}$$



Equilibrium means steady state  
[assume horizontally homogeneous]

$$0 = \partial F_q / \partial p + \omega \partial q^- / \partial p$$

$$0 = \partial F_\theta / \partial p + \omega \partial \theta^- / \partial p + \partial \theta_{\text{Rad}} / \partial t$$

[where  $F_q$  and  $F_\theta$  represent the convective fluxes of total water and 'liquid water potential temperature' above cloud-base]

Integrate to give fluxes from  $\omega$ ,  $\theta$  and  $q$  profiles, and  $\partial \theta_{\text{Rad}} / \partial t$ .

This gives *equilibrium fluxes* [in units of  $\text{W m}^{-2}$ ] *from profiles*

## Simple mass-flux model [illustration]

Can couple fluxes with a *mass flux transport model* for shallow convection

$$F_q = \Omega_q (q_c - q) \text{ with } q_c = q_B \text{ a cloud-base value of } 16.54 \text{ gkg}^{-1}$$

and compute the  $\Omega_q$  shown in the figure.

# Shallow Cumulus

- non-precipitating
- net  $LH = 0$
- but transport heat because condense water, advect it upward and reevaporate it [a “refrigerator”]
- buoyant, because of condensation but still ‘cold’, because of liquid
- conserved variables:  $\theta_E = \theta + Lq/C_p$   
 $\theta_L = \theta - Ll/C_p$   
 $q_T = q + \ell$



- represent by mass transport of air with sub-cloud properties to higher levels
- equilibrium structure over ocean is balance of convective transports, subsidence, and radiative flux divergence (cooling)

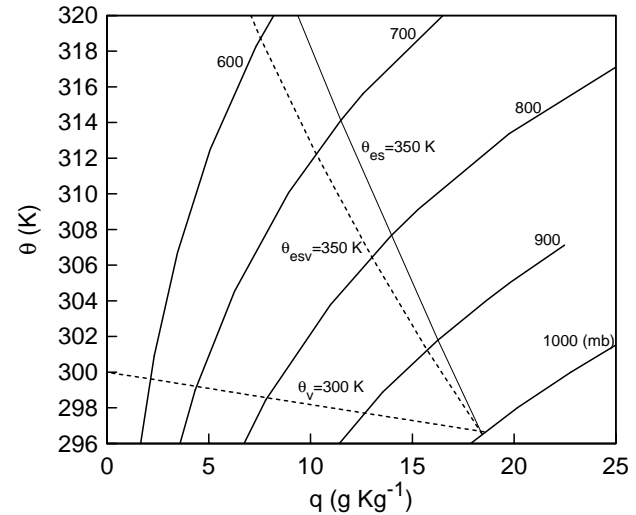
## Conserved Variable diagram – 2

- Similar to other thermodynamic diagrams; just  $\theta$ ,  $q$  as axes

Dry virtual potential temperature

$$\theta_v = \theta(1 + .608 * q / 1000)$$

- vapor is less dense
- SP of equal density
- Slopes 1K every  $6 \text{ g kg}^{-1}$   
[Could use as axis]



Wet virtual potential temperature

- if parcels carry liquid .. Denser;  $\Delta \ell = 2 \text{ g kg}^{-1} / 100 \text{ hPa}$

$$\theta_v = \theta(1 + .608 * q / 1000 - \ell / 1000)$$

- line of equal density

$$(\partial \theta / \partial p)_{\theta_{esv}} \approx 0.9 (\partial \theta / \partial p)_{\theta_{es}}$$

## Parameterizing shallow convection with a mixing line representation

- parameterize a cloud field: what do these simple diagnostic studies tell us?
- two approaches:

a) parameterize fluxes, and their gradients:

eg with mass flux model; say cloud-base  $q$ -flux = surface  $q$  flux

[Problem from a ‘climate perspective is that system may drift to either dry or cloudy state]

b) parameterize structure: eg ‘mixed layer’ or ‘mixing line’.

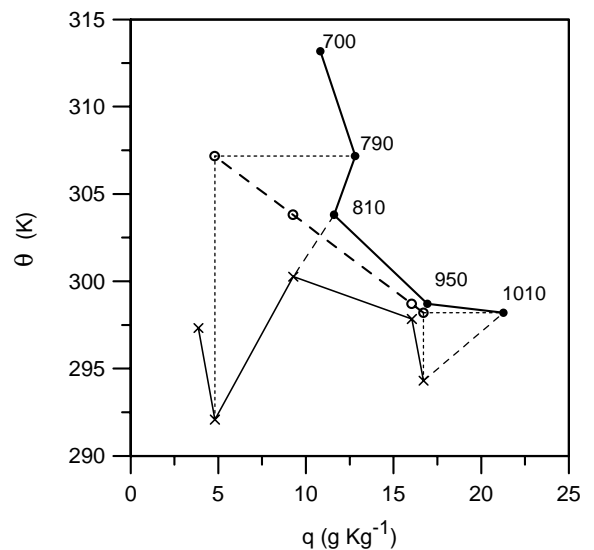
Single mixing line can represent whole BL structure of both clear and cloudy air.

Unsaturated air: find  $T$ ,  $T_d$  at  $p$  by drawing lines of constant  $\theta$  and  $q$

Cloudy air: find  $T$ ,  $T_d$  [for total water] at  $p$  by drawing lines of constant  $\theta_{es}$  and  $q$

A type of convective adjustment.

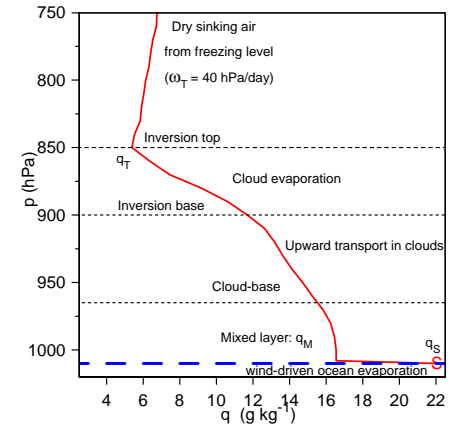
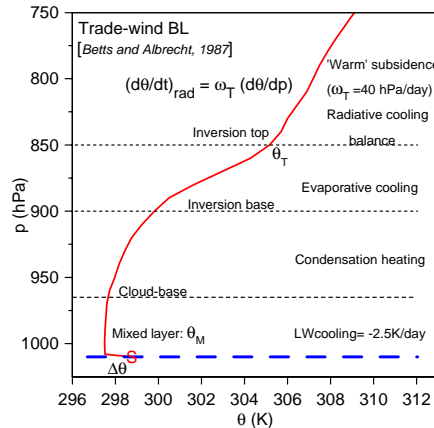
[example: advanced students read paper/Betts and Ridgway,1989]



# How does ocean BL and land differ?

Radiative cooling  
SH, subsidence

Evaporation and  
subsidence



Stays a little cooler than ocean and sub-saturated:  
surface wind and subsidence control evaporation  
[ocean store suns heat; diurnal cycle small]

## LAND: what are the essential differences??

Sun heats surface and drives large diurnal cycle; daytime unstable;  
cools radiatively at night; at night stable BL

Surface not saturated.. Except inside leaves.

Sun drives evaporation through photosynthesis  
[coupled to  $CO_2$  uptake]

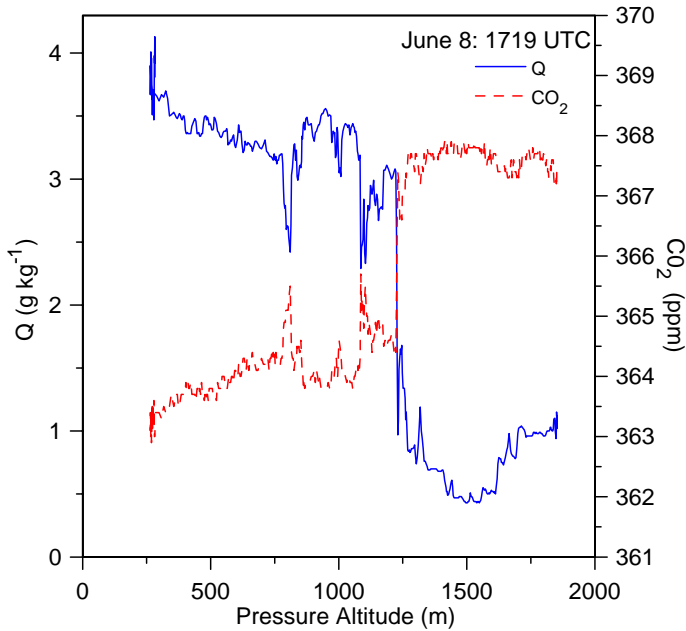
Subsidence of dry air still plays key role, averaged over 24hrs.

*Need to understand mean state and diurnal cycle*

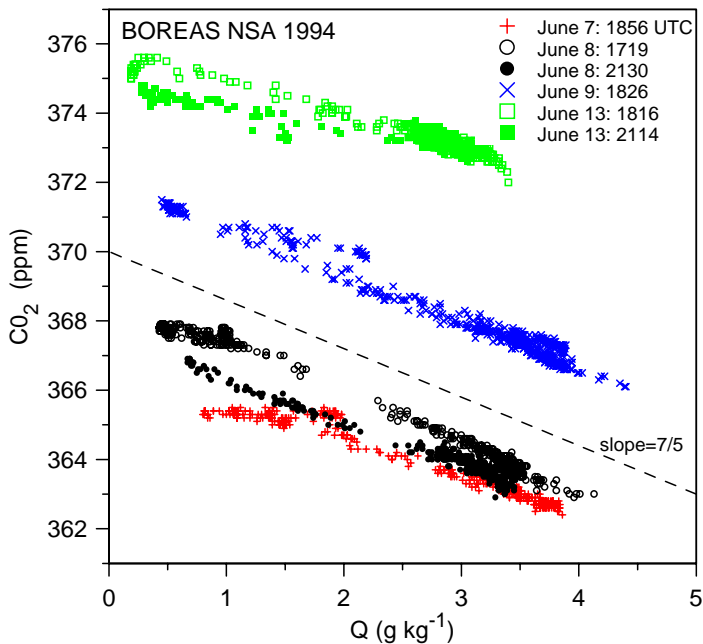


# Coupling of CO<sub>2</sub> and water vapor through the BL

BOREAS Northern Study area [Thompson, Manitoba]



**Figure 1** Coupling of CO<sub>2</sub> and water vapor profiles of June 8 at 1719 UTC (LST=UTC-6h)



**Figure 2.** Profiles through the mixed layer on four days in June, showing tight coupling between water vapor and CO<sub>2</sub> structure. Illustrative slope of 7 ppm CO<sub>2</sub> to 5 g kg<sup>-1</sup> is shown.

# RH, LCL and pressure height of cloud-base are fundamental BL quantities

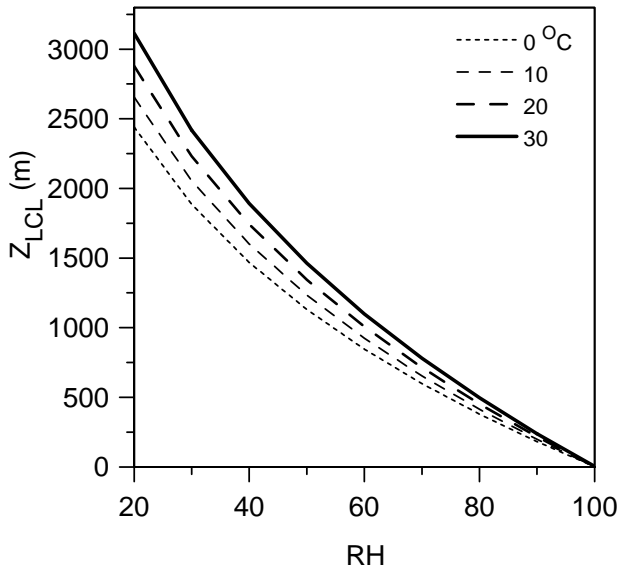


Fig. 19a.. Relation between height of cloudbase and RH as surface temperature varies.  
[Note independent of surface pressure]

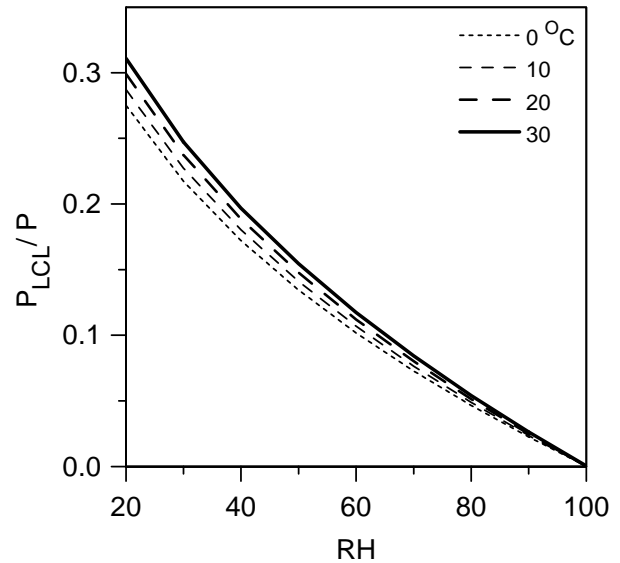


Fig.19b. As Fig. 4a for ratio of  $P_{LCL}$  to surface pressure  $p$   
[Note dependence on  $T$  is weak]

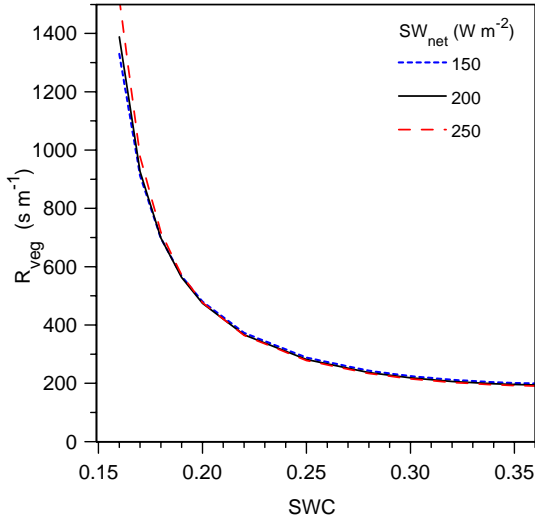
## – Over land, there is link to evaporative resistance

Dry soils → large resistance to evaporation

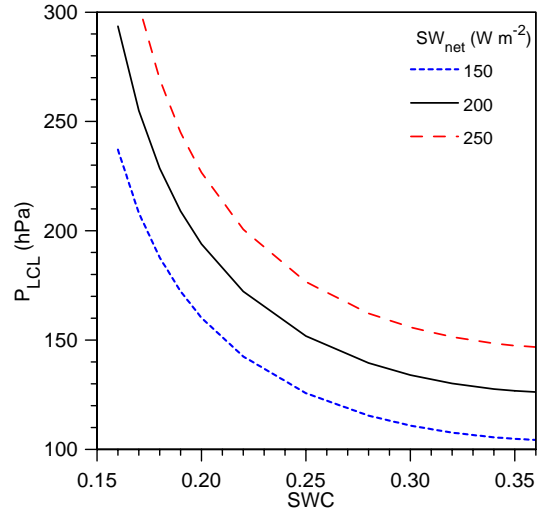
Extra resistance produces drop of saturation from inside leaf to outside leaf

Reduces relative humidity (RH) and increases BL depth

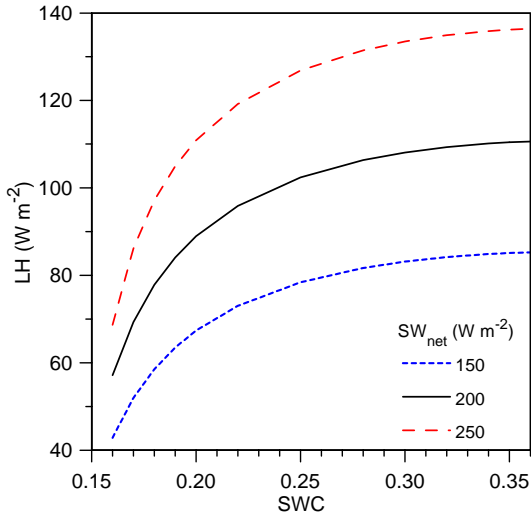
**We can create an “equilibrium model” by averaging over 24hour cycle**  
 – how does mean BL depth and fluxes depend on soilwater and solar forcing?



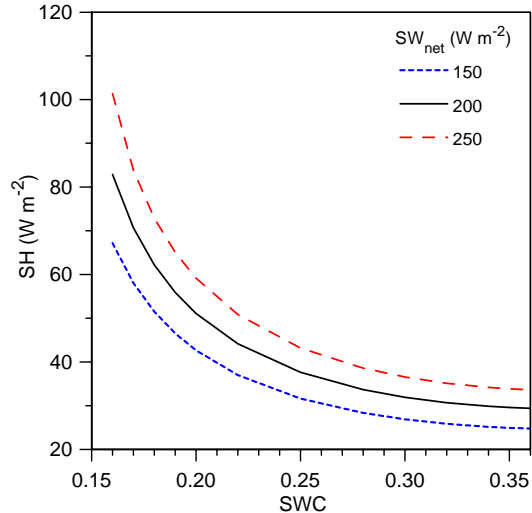
**Figure 20.** Dependence of stomatal resistance on SWC, and  $SW_{net}$



**Figure 21.** Dependence of BL depth on SWC and  $SW_{net}$



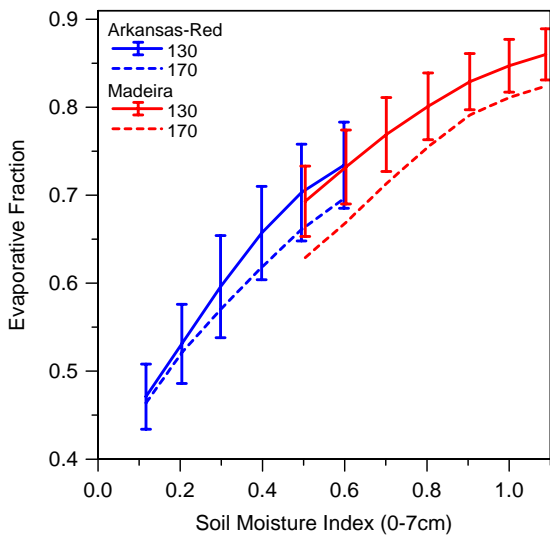
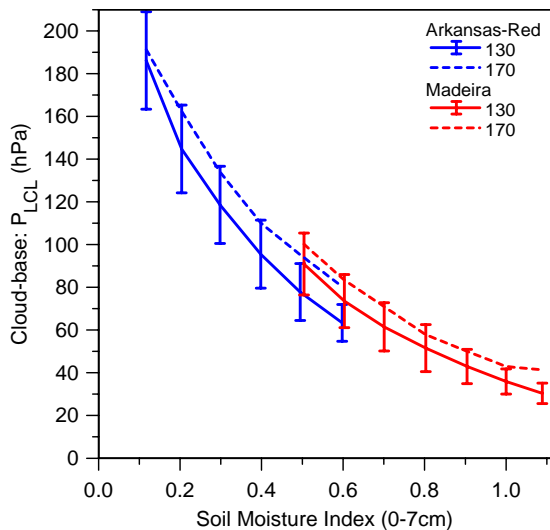
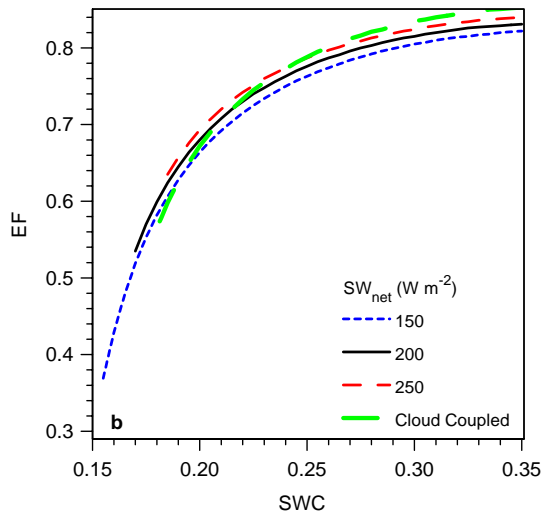
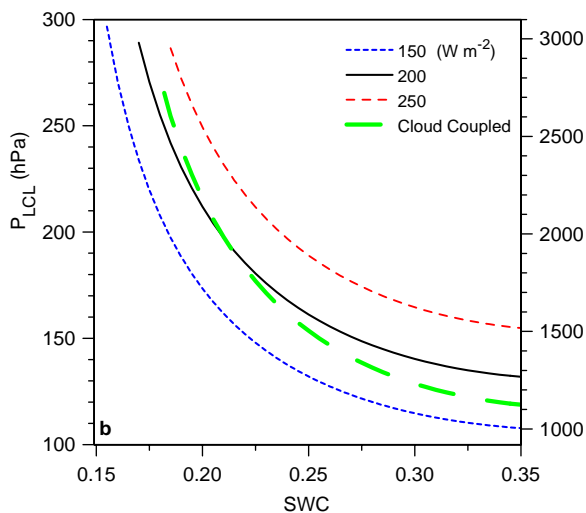
**Figure 22.** Latent heat on BL depth



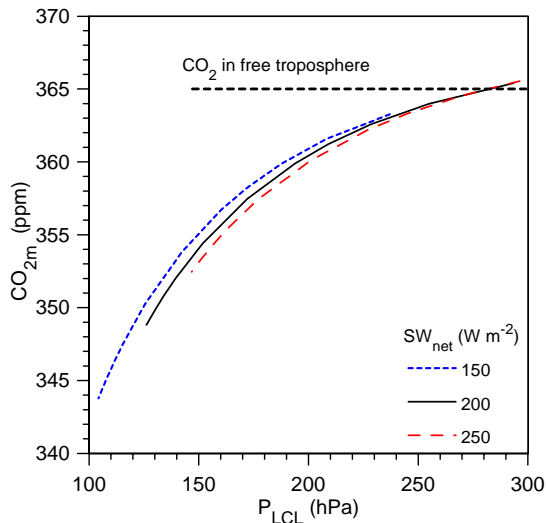
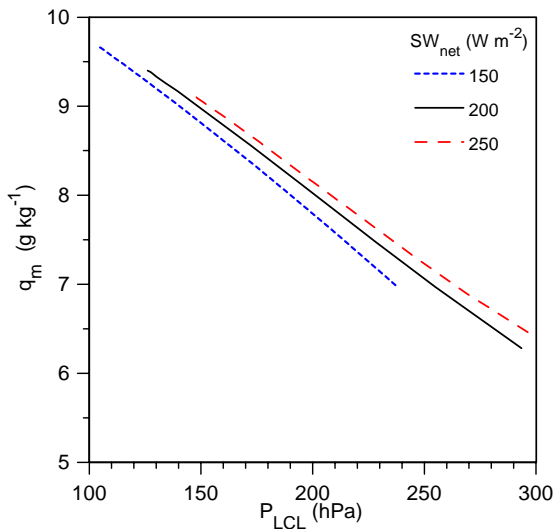
**Figure 23.** Sensible heat on BL depth

[see Betts: *J. Hydrometeorol.* 2000]

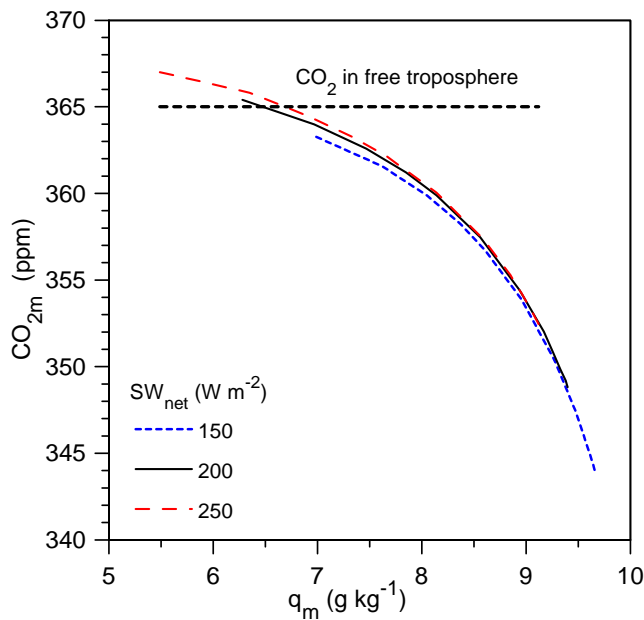
# Compare idealized model and diurnally averaged ERA40



# Evaporation and photosynthesis are linked to *same* vegetative resistance



**q and CO<sub>2</sub> at equilibrium are functions of BL depth**



**CO<sub>2</sub> and water vapor are tightly coupled**

# **Coupling between CO<sub>2</sub>, water vapor, temperature and radon and their fluxes in an idealized equilibrium boundary layer over land.**

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- Brent Helliker and Joe Berry

Carnegie Inst. of Washington, Stanford, CA

[*J. Geophys. Res.* in revision]

# Idealized equilibrium boundary layer over land

- Extension of Betts (2000) in *J. Hydromet.*
- Idealization is to average over diurnal cycle
- **Extensions**
  - add vegetation model
  - CBL and ML equilibrium
  - Simple coupling of radiation to clouds

# Purpose

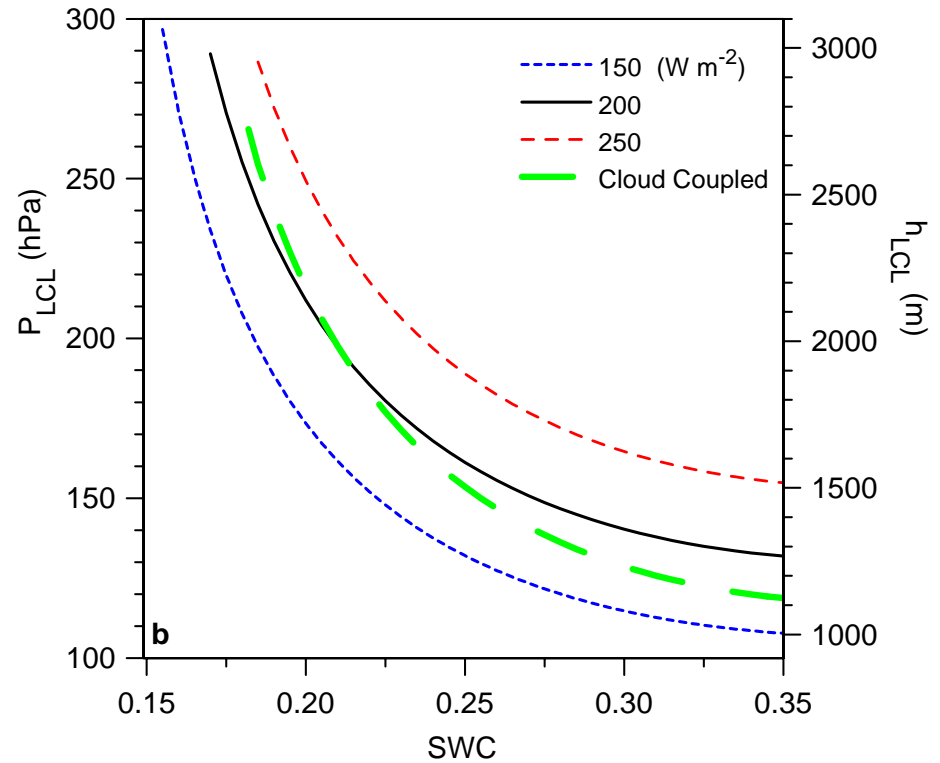
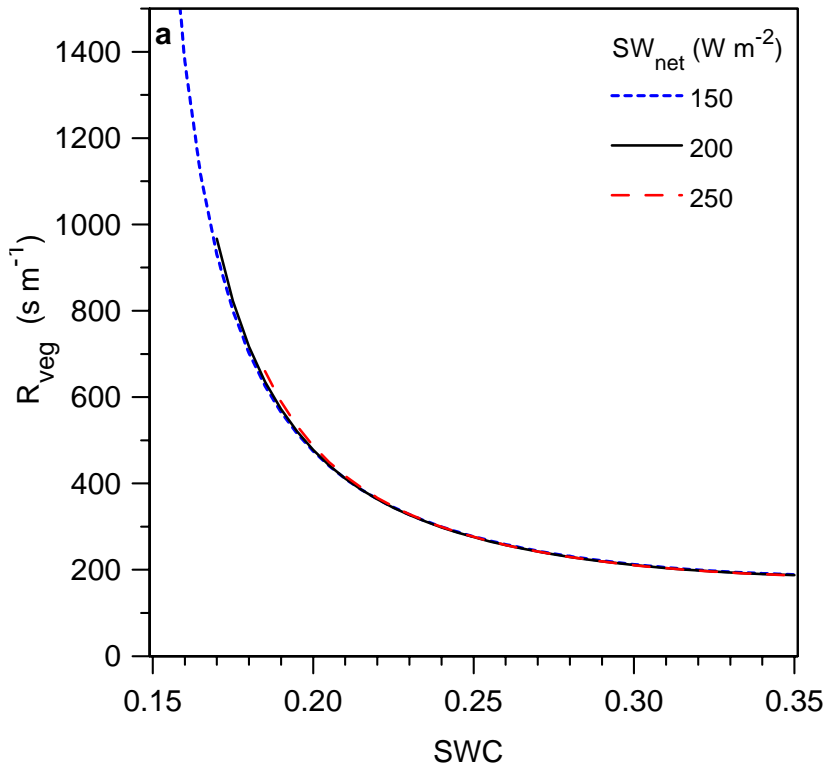
- Couple mixed layer 'equilibrium' of potential temperature, water vapor, CO<sub>2</sub> (and radon) with the surface energy and water budgets and net ecosystem exchange (and surface radon flux)
- Suggest that regional ML budgets may give useful constraints on regional carbon budgets.



# Idealized equilibrium model

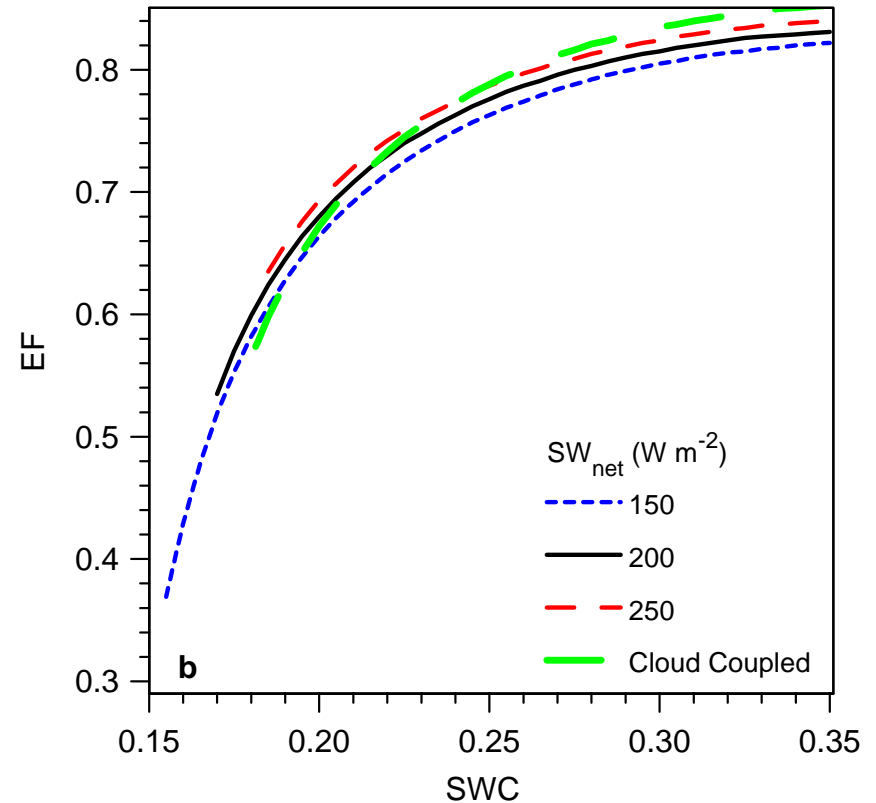
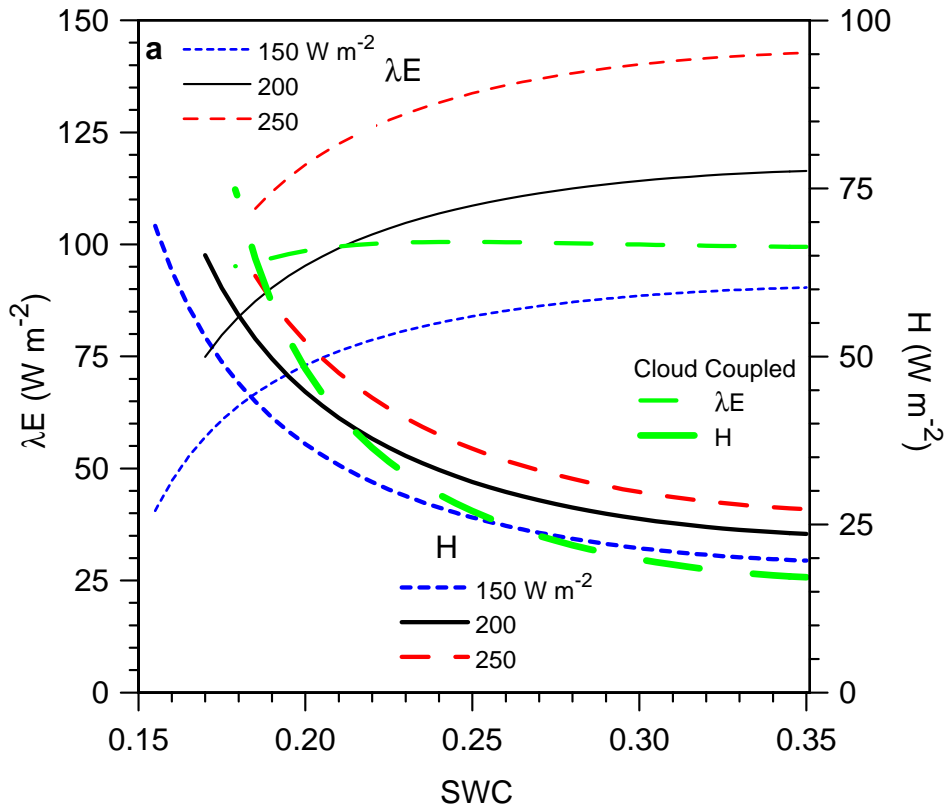
- Surface energy budget: diurnal mean
- Radiative fluxes coupled through cloud cover
- Photosynthetic controlled evaporation, linked to stomatal resistance, calculated from Ball-Berry model, fitted to Wisconsin tall tower data.
- ML and CBL equilibrium
- 45 equations ... read the paper

# Equilibrium solutions



- Sensitivity of *vegetative resistance* and *ML depth* to soil water and net short-wave

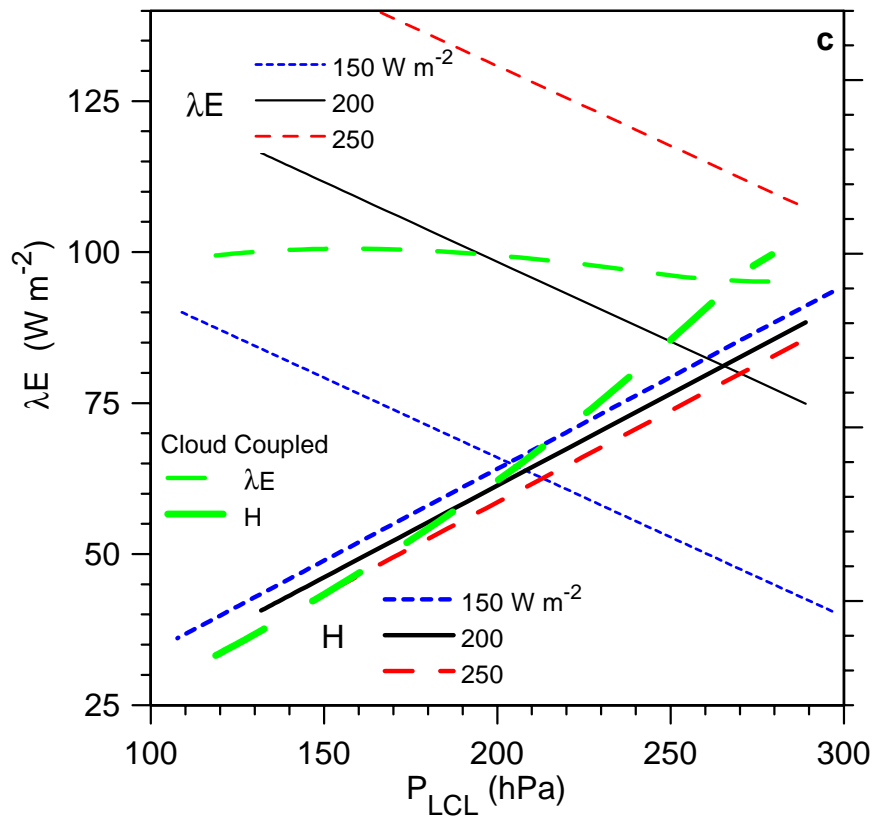
# Surface energy fluxes and EF as a function of soil water content



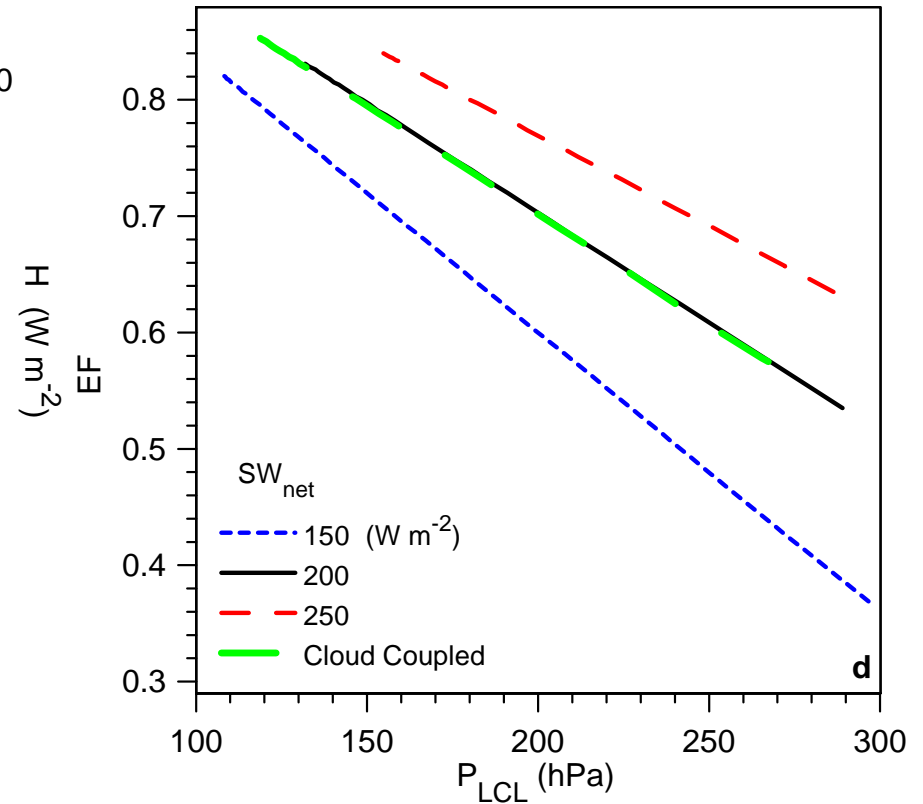
Latent and sensible heat fluxes

Evaporative fraction

# Surface energy fluxes and EF as a function of ML depth: $P_{LCL}$

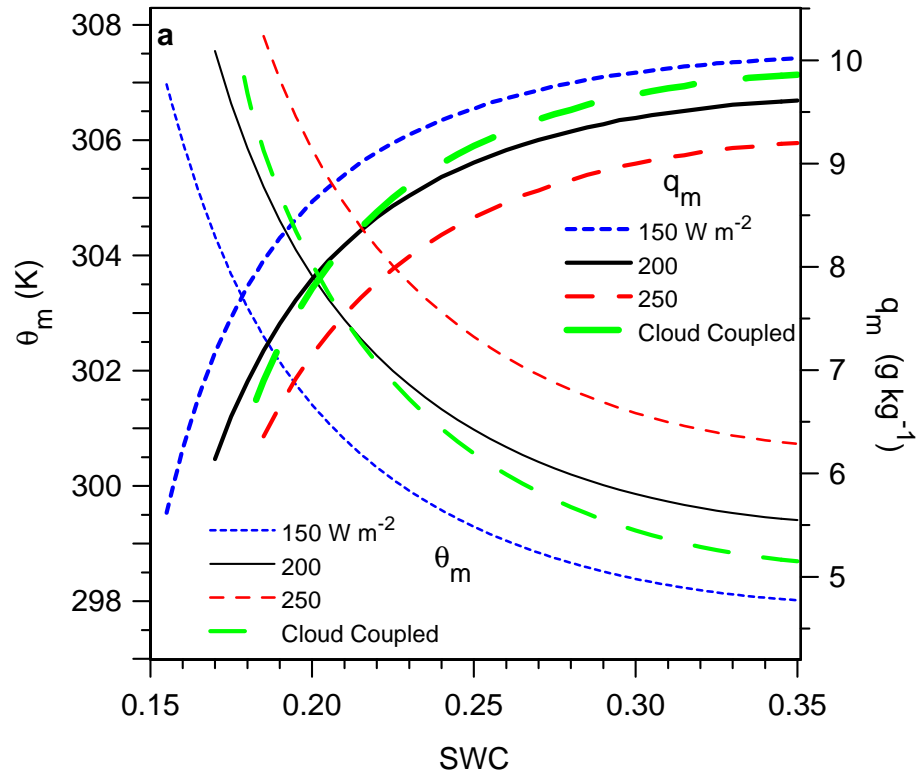


Latent and sensible heat fluxes

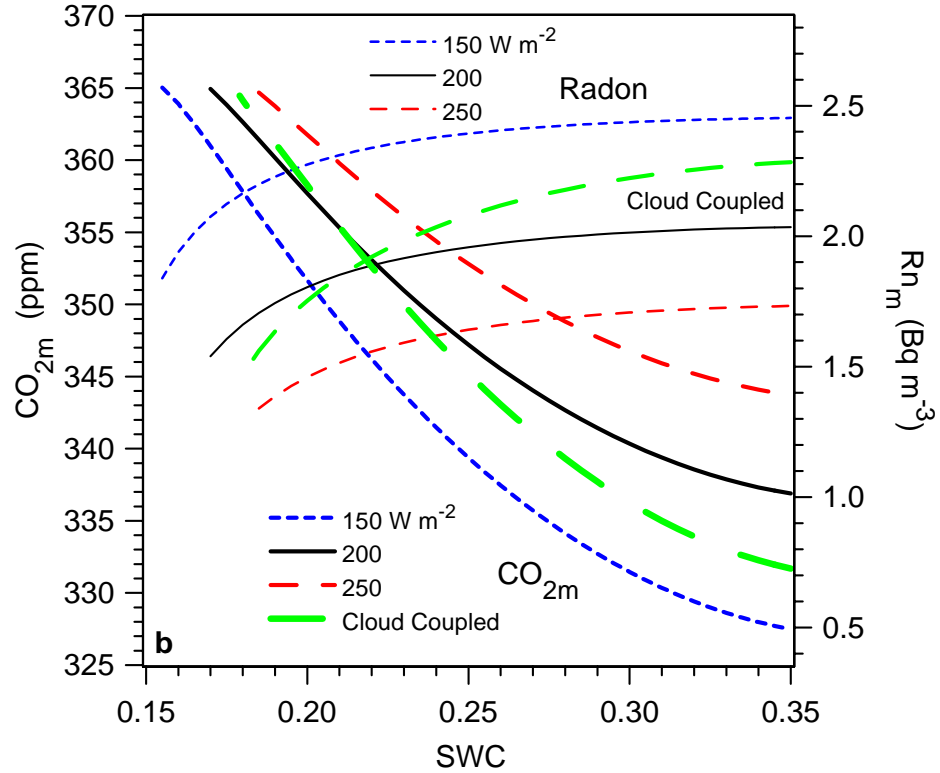


Evaporative fraction

# ML properties as a function of soil water content

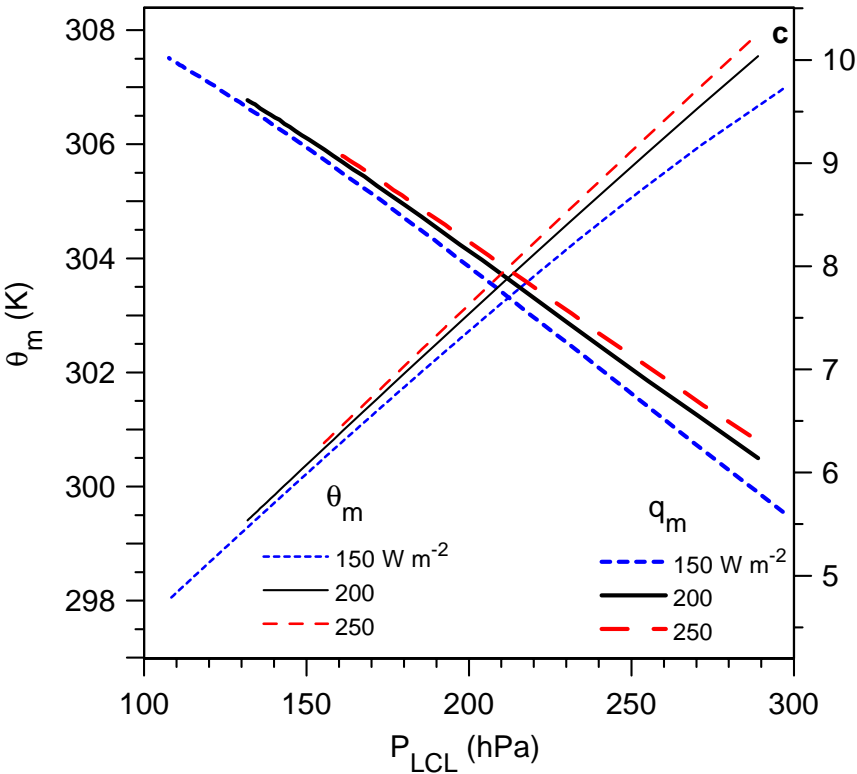


ML potential temperature  
and mixing ratio

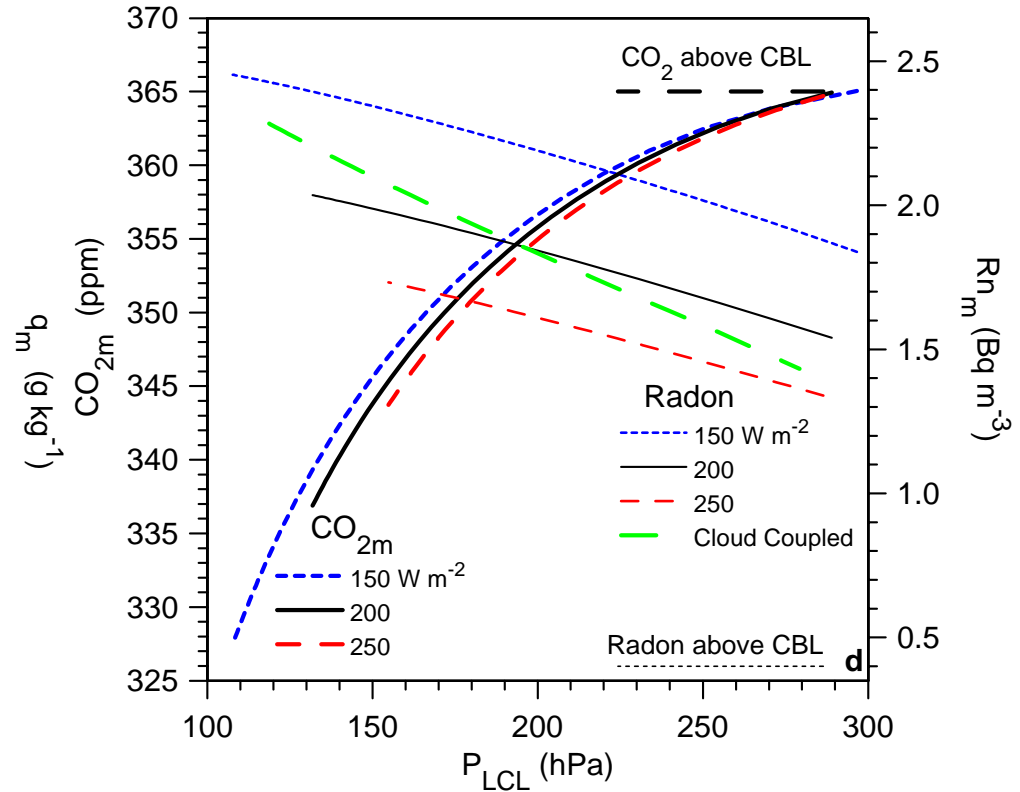


ML CO<sub>2</sub> and radon

# ML properties as a function of ML depth: $P_{LCL}$



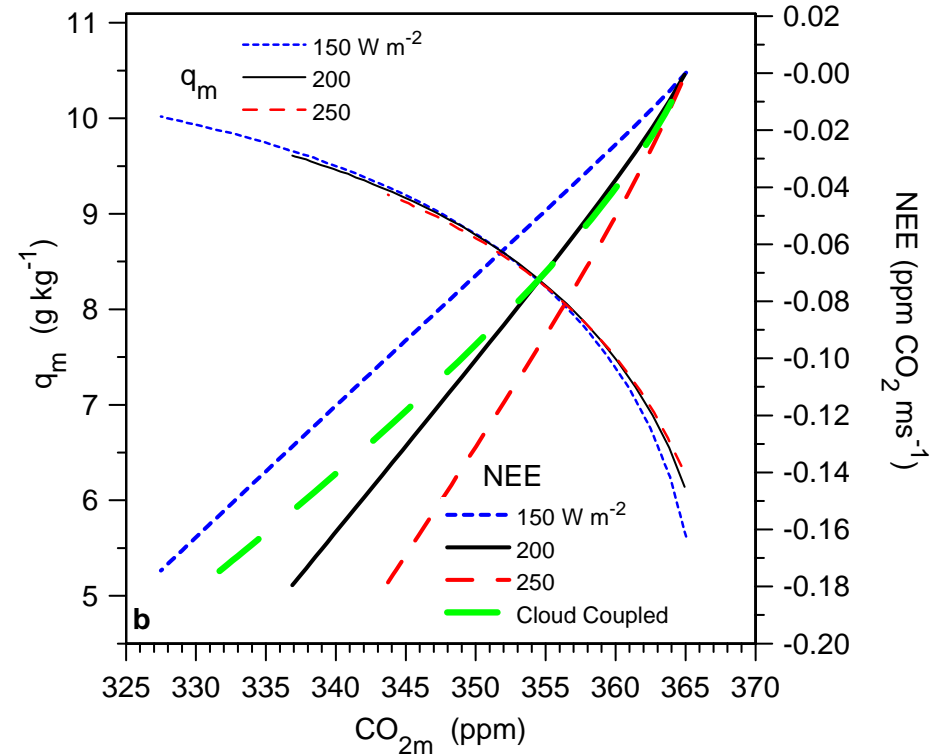
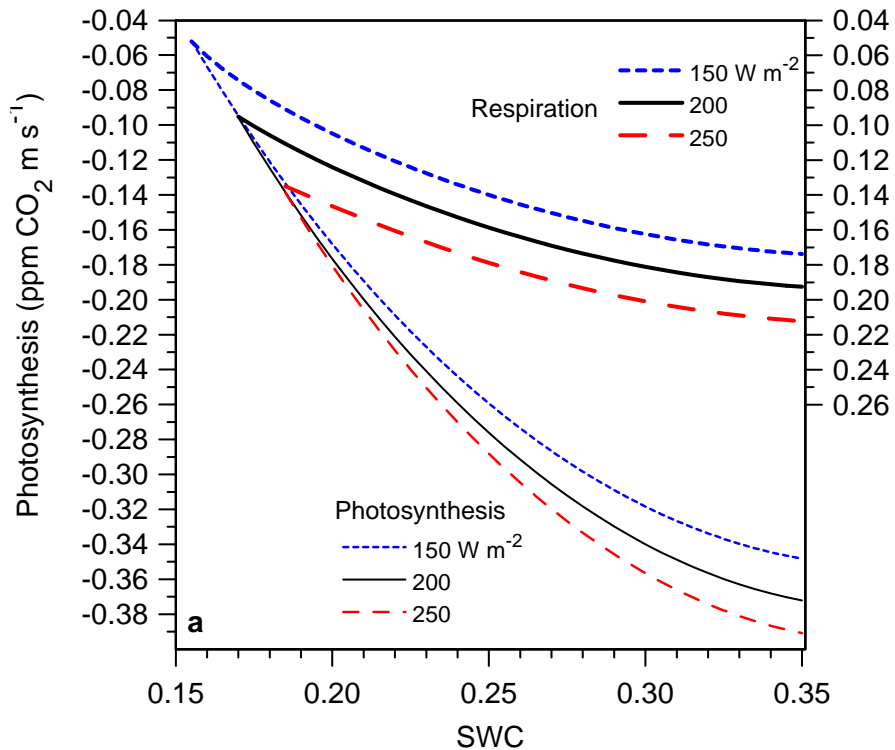
ML potential temperature  
and mixing ratio



ML  $CO_2$  and radon

# Photosynthesis and respiration

## Coupling of $\text{CO}_{2m}$ to $q_m$ and NEE



Photosynthesis; respiration

$\text{CO}_{2m}$  against  $q_m$  and NEE

# Conclusions-1

- SWC is primary control on NEE and on evaporation through stomatal resistance
- Dry soil: equilibrium depth of the ML increases sharply, as reduced evaporation leads to a warmer drier equilibrium
- LCL is powerful constraint on ML depth
- Radiative impact of clouds on equilibrium



# Conclusions-2

- Two different perspectives:
  - as a function of SWC
  - as function of cloud-base height
- Important coupling between ML  $q$  and  $\text{CO}_2$ , and between NEE exchange and  $\text{CO}_2$ 
  - useful for carbon budget estimates

*Preprint at [ftp://members.aol.com/bettspapers/BHB\\_JGR.pdf](ftp://members.aol.com/bettspapers/BHB_JGR.pdf)*

# Take away these ideas

*Ocean equilibrium*: balance of radiative cooling, subsidence and surface fluxes

giving a typical tradewind BL with cloud-base 50hPa above surface and a 150hPa deep shallow cumulus layer....

[Solar heating absorbed in deep ocean mixed layer]

*Land diurnal cycle* driven by solar heating, but *equilibrium* similar to ocean, except a drier mean state because additional ‘vegetative’ resistance to evaporation at surface.

*CO<sub>2</sub> and water vapor coupled in BL over vegetation.*