

# Boundary layer equilibrium

## – over tropical oceans

[2005]

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Based on:

Betts, A.K., 1997: Trade Cumulus: Observations and Modeling. Chapter 4 (pp 99-126) in “*The Physics and Parameterization of Moist Atmospheric Convection*, Ed. R. K. Smith, NATO ASI Series C: Vol. **505**, Kluwer Academic Publishers, Dordrecht, 498pp.

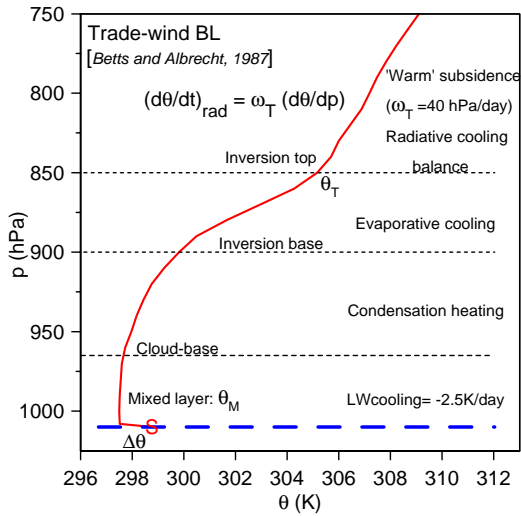
Betts, A. K., and B. A. Albrecht, 1987: Conserved variable analysis of boundary layer thermodynamic structure over the tropical oceans. *J. Atmos. Sci.*, **44**, 83-99.

Betts, A. K. and W. Ridgway, 1988 : Coupling of the radiative, convective and surface fluxes over the equatorial Pacific. *J. Atmos. Sci.*, **45**, 522-536.

Betts, A.K. and W. L. Ridgway, 1989: Climatic equilibrium of the atmospheric convective boundary layer over a tropical ocean. *J. Atmos. Sci.*, **46**, 2621-2641.

Betts, A. K., 2000: Idealized model for equilibrium boundary layer over land. *J. Hydrometeorol.*, **1**, 507-523.

Betts, A. K., B. Helliker and J. Berry, 2004, Coupling between CO<sub>2</sub>, water vapor, temperature and radon and their fluxes in an idealized equilibrium boundary layer over land. *J. Geophys. Res.* **109**, D18103, doi:10.1029/2003JD004420.



Why is mixed layer cooler than the ocean SST?

LW cooling = -2.5 K/day

Clouds redistribute heat and water and modify radiative balance

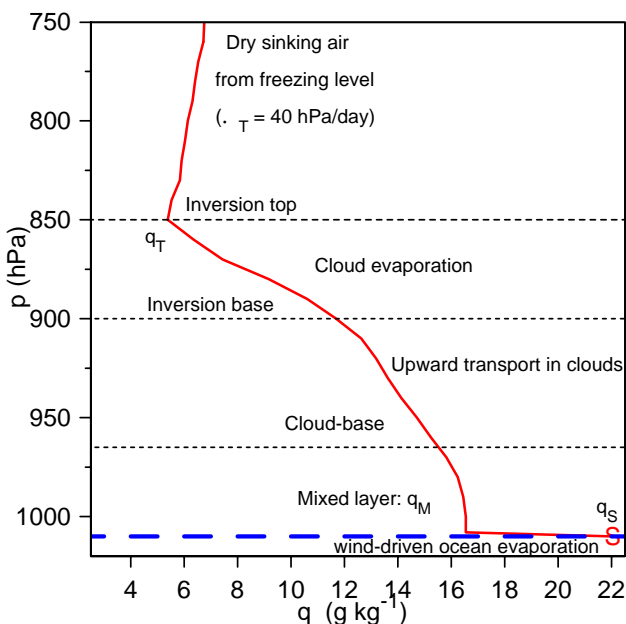
Equilibrium for whole layer:

$$0 = (g/C_p) \Delta N_T + \omega_0 \Delta\theta + \omega_T (\theta_T - \theta_M)$$

$$\begin{array}{ccc} -40 & +10 & +30 \text{ W m}^{-2} \\ \text{cooling} & \text{surface flux} & \text{subsidence} \end{array}$$

Surface velocity scale:  $\omega_0 = \rho V_0 C_D \approx 90 \text{ hPa/day}$

Subsidence:  $\omega_T \approx 40 \text{ hPa/day}$



Why is the mixed layer not saturated,  
as the air blows over ocean?

Evaporation from ocean is balanced  
by subsidence of dry air above.

$$0 = \omega_0 (q_S(\text{SST}) - q_M) + \omega_T (q_T - q_M)$$

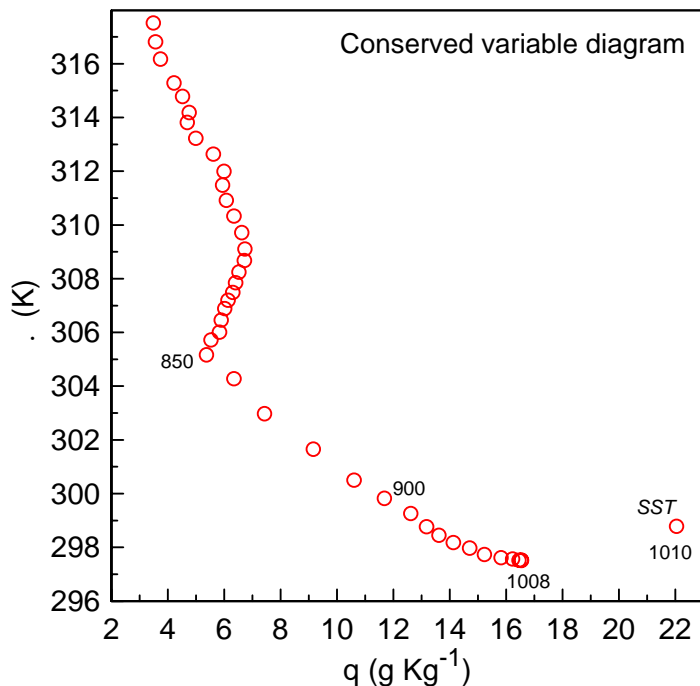
$$q_M = [\omega_0 q_S(\text{SST}) + \omega_T q_T] / (\omega_0 + \omega_T)$$

A weighted average

$$q_M = [90 \cdot 22 + 40 \cdot 5] / 130 = 16.7 \text{ g/kg}$$

$$\text{so } \theta_{EM} \approx 346\text{K}$$

$$\text{cloud-base} \approx 960\text{hPa}$$



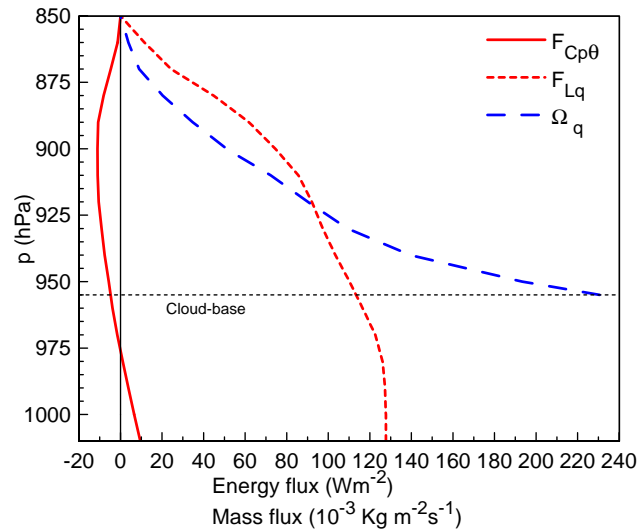
Can think of the two balances on  
a ‘conserved parameter’ diagram:  
“Mixing” of surface point and  
850hPa point, modified by  
radiation.

## Relate equilibrium structure to convective fluxes: $F_q$ , $F_\theta$ [illustration]

Assume  $\omega = 40\text{hPa/day}$  in cloud layer, below cloud-base decreases linearly to zero at surface.

Assume radiative cooling

$$\partial\theta_{\text{Rad}}/\partial t = -2.4 \text{ K/day}$$



Equilibrium means steady state  
[assume horizontally homogeneous]

$$0 = \partial F_q / \partial p + \omega \partial q^- / \partial p$$

$$0 = \partial F_\theta / \partial p + \omega \partial \theta^- / \partial p + \partial \theta_{\text{Rad}} / \partial t$$

[where  $F_q$  and  $F_\theta$  represent the convective fluxes of total water and 'liquid water potential temperature' above cloud-base]

Integrate to give fluxes from  $\omega$ ,  $\theta$  and  $q$  profiles, and  $\partial \theta_{\text{Rad}} / \partial t$ .

This gives *equilibrium fluxes* [in units of  $\text{W m}^{-2}$ ] *from profiles*

## Simple mass-flux model [illustration]

Can couple fluxes with a *mass flux transport model* for shallow convection

$$F_q = \Omega_q (q_c - q) \text{ with } q_c = q_B \text{ a cloud-base value of } 16.54 \text{ gkg}^{-1}$$

and compute the  $\Omega_q$  shown in the figure.

# Shallow Cumulus

- non-precipitating
- net latent heat release = 0
- but transport heat because condense water, advect it upward and reevaporate it [a “refrigerator”]
- buoyant, because of condensation but still ‘cold’, because of liquid
- conserved variables:  $\theta_E = \theta + Lq/C_p$   
 $\theta_L = \theta - L\ell/C_p$   
 $q_T = q + \ell$



- represent by mass transport of air with sub-cloud properties to higher levels
- equilibrium structure over ocean is balance of convective transports, subsidence, and radiative flux divergence (cooling)

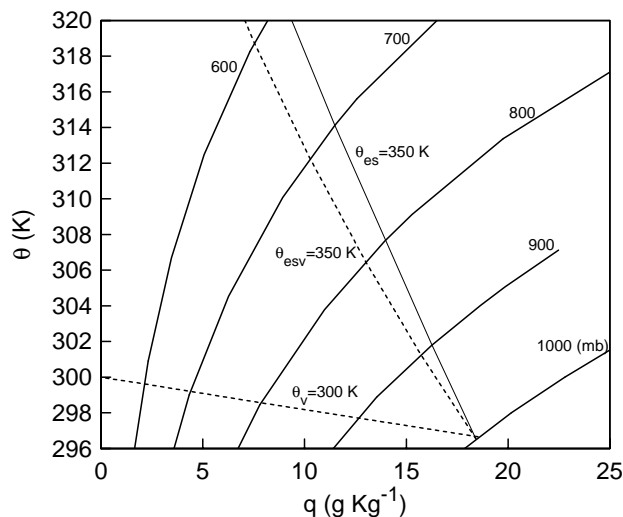
## Conserved Variable diagram – 2

- Similar to other thermodynamic diagrams; just  $\theta$ ,  $q$  as axes

Dry virtual potential temperature

$$\theta_v = \theta(1 + .608 * q / 1000)$$

- vapor is less dense
- 'Saturation Points' of equal density
- Slopes 1K every  $6 \text{ g kg}^{-1}$   
[Could use as axis]



Wet virtual potential temperature

- if parcels carry liquid .. Denser;  $\Delta \ell = 2 \text{ g kg}^{-1} / 100 \text{ hPa}$
- $\theta_v = \theta(1 + .608 * q / 1000 - \ell / 1000)$
- line of equal density  
 $(\partial \theta / \partial p)_{\theta_{\text{esv}}} \approx 0.9 (\partial \theta / \partial p)_{\theta_{\text{es}}}$

## Parameterizing shallow convection with a mixing line representation

- parameterize a cloud field: what do these simple diagnostic studies tell us?
- two approaches:

a) parameterize fluxes, and their gradients:

eg with mass flux model; say cloud-base q-flux = surface q flux

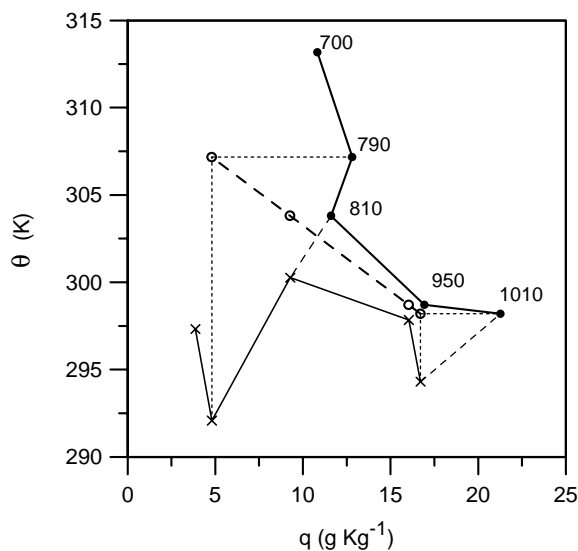
[Problem from a ‘climate perspective is that system may drift to either dry or cloudy state]

b) parameterize structure: eg ‘mixed layer’ or ‘mixing line’.

Single mixing line can represent whole BL structure of both clear and cloudy air.

Unsaturated air: find  $T$ ,  $T_d$  at  $p$  by drawing lines of constant  $\theta$  and  $q$

Cloudy air: find  $T$ ,  $T_d$  [for total water] at  $p$  by drawing lines of constant  $\theta_{es}$  and  $q$



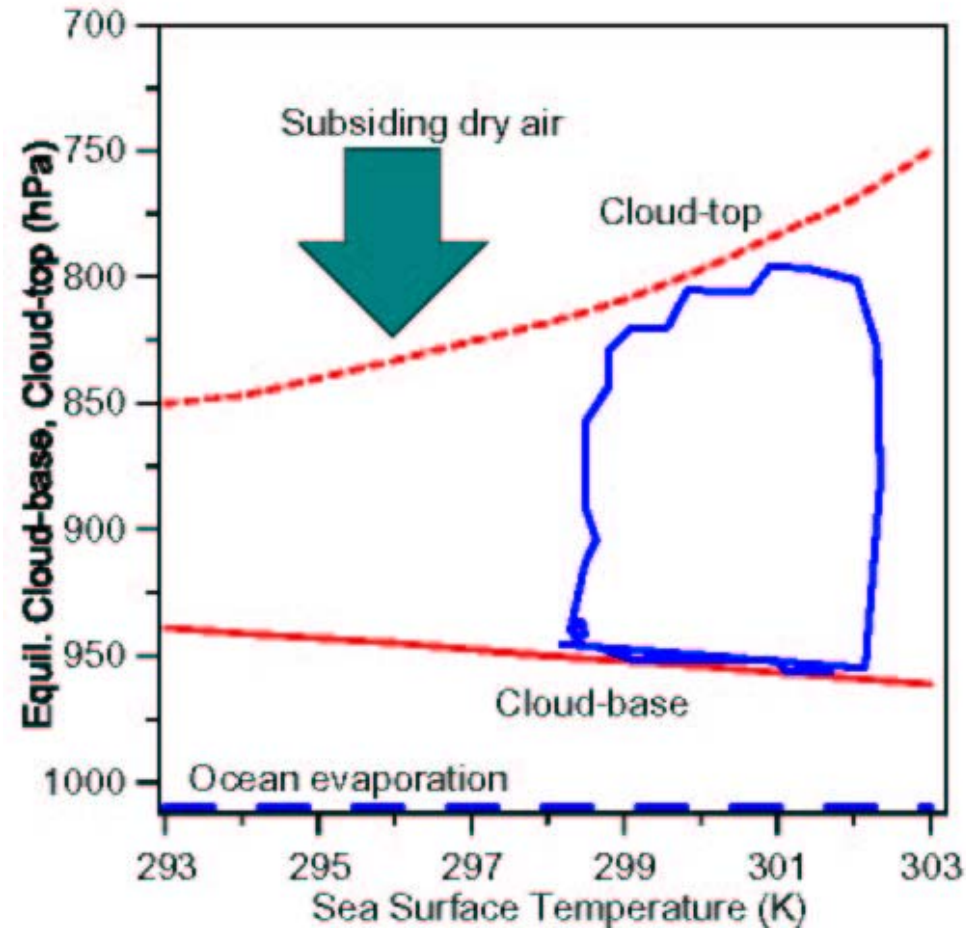
Often useful to have a model for structure; eg to compute radiation

[example: Betts and Ridgway, 1989]

# Climate equilibrium in the Tropics



Shallow Trade-wind cumulus  
flowing into deep precipitating  
tropical convergence zone



Trade cumulus balance  
between ocean evaporation  
and sinking of dry air



# **Tropical Climate equilibrium**

*[Betts and Ridgway, JAS 1988, 1989]*

- **Consider subsiding branches of tropical circulation, like the Trades.**

**[Moisture evaporated here flows into the convergence zones and tropical disturbances where it is precipitated]**

- **Energy balance closures give radiative-convective equilibrium; but there are several important timescales**

# 1. Subcloud layer thermal balance [one-day timescale]

$$H + H_B = \Delta N_B$$

Where radiative cooling of sub-cloud layer,

$$\Delta N_B \approx 10 \text{ Wm}^{-2} \text{ } [-2.5 \text{ K/day} * 50 \text{ hPa}]$$

Cloud-base flux:  $H_B \approx -0.2 H$  [surface flux]

$$\text{Giving } H = \Delta N_B / 1.2 \approx 8 \text{ Wm}^{-2}$$

So radiative cooling of sub-cloud layer gives small sea-air temperature difference and small sensible heat flux.

[Bowen ratio over tropical oceans is small.]

## 2. CBL budgets

[1 to 2-day timescale]

### Heat

$$\underset{\substack{\text{surface} \\ \text{cooling}}}{H} + \underset{\substack{\text{subsidence warming}}}{(C_p/g)\omega_T(\theta_T - \theta_M)} = \underset{\substack{\text{CBL radiative}}}{\Delta N_T}$$

### Water

$$\underset{\substack{\text{surface}}}{\lambda E} + \underset{\substack{\text{subsidence drying}}}{(L/g)\omega_T(q_T - q_M)} = 0$$

$\lambda E$  linked to subsidence  $\omega_T$

Given  $\omega_T$ ,  $(\Delta N_T - H)$  gives  $\theta_T$  and CBL-top

### 3. Tropospheric energy balance [10-day timescale]

The atmospheric energy balance averaged over the tropics can be written

$$H + \lambda E = \Delta N_{\text{TR}} + \text{atmospheric export from tropics}$$

Where radiative cooling of troposphere  $\Delta N_{\text{TR}} \approx 150 \text{ Wm}^{-2}$

So surface evaporation

$$\lambda E \approx 150 - 8 \approx 142 \text{ Wm}^{-2}$$

The mechanism is that the radiative cooling drives the subsiding branch, bringing dry air into the CBL, which balances evaporation locally; and the moisture flows into the convergence zones, condensing and releasing latent heat which balance the radiative cooling.

## 4. Ocean mixed layer and SST equilibrium [ $>100$ day timescale]

$$H + \lambda E = N + \text{oceanic export from tropics}$$

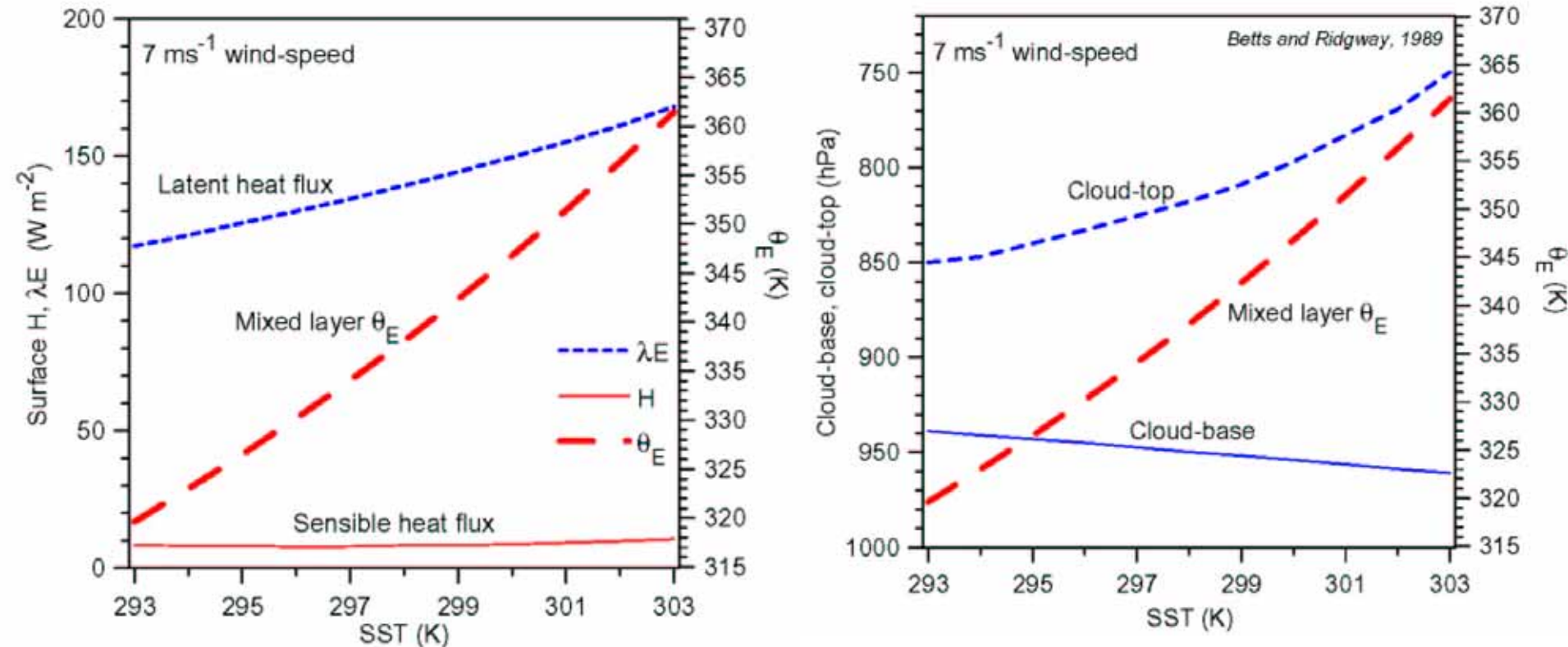
Where  $N$  is the net incoming radiative flux at the surface [shortwave + longwave]

This is the long-time-scale equilibrium that controls SST. [The big terms are the shortwave heating and the evaporation, but the downward longwave flux depends on water vapor 'greenhouse effect']

Solve the coupled system using 1 and 2 [plus 3, 4].

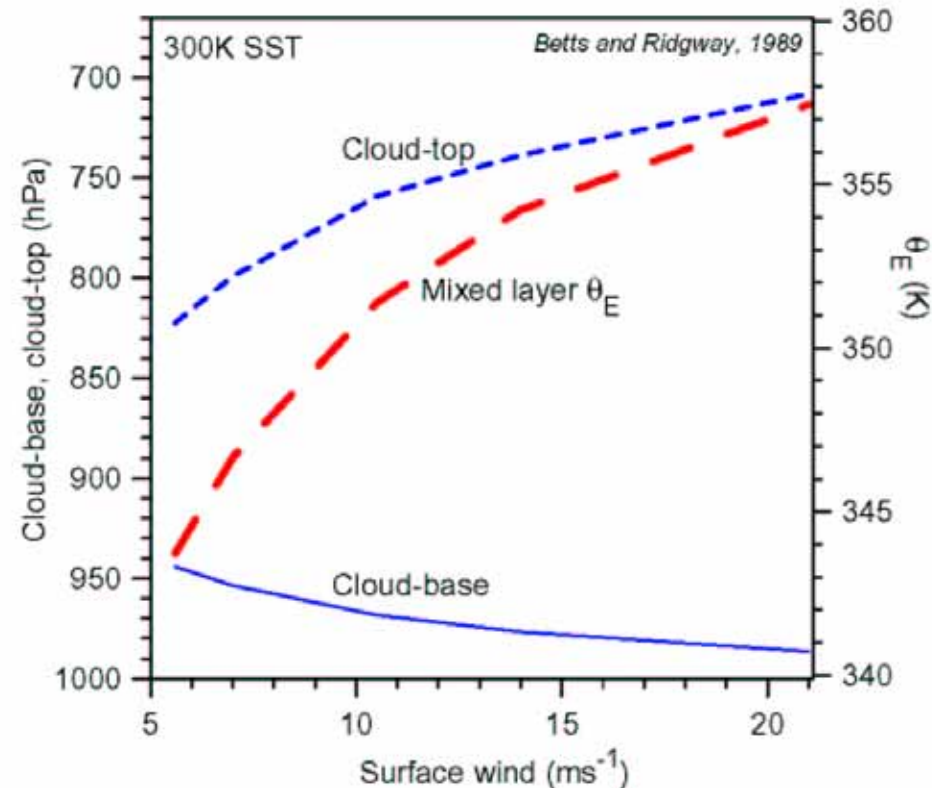
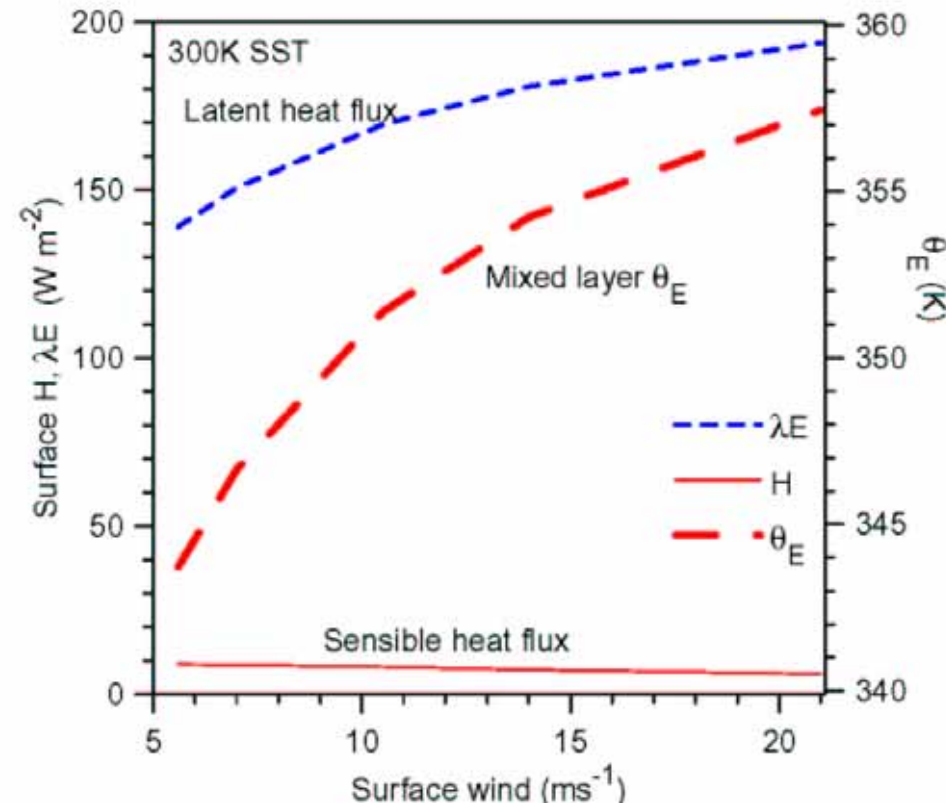
*[Betts and Ridgway, 1989]*

# Vary SST with fixed wind-speed



- Evaporation increases with SST *Uses 1,2,3*
- $\theta_E$  increases with SST [and cloud-base descends a little]

# Vary wind-speed with fixed SST; $\omega_T$

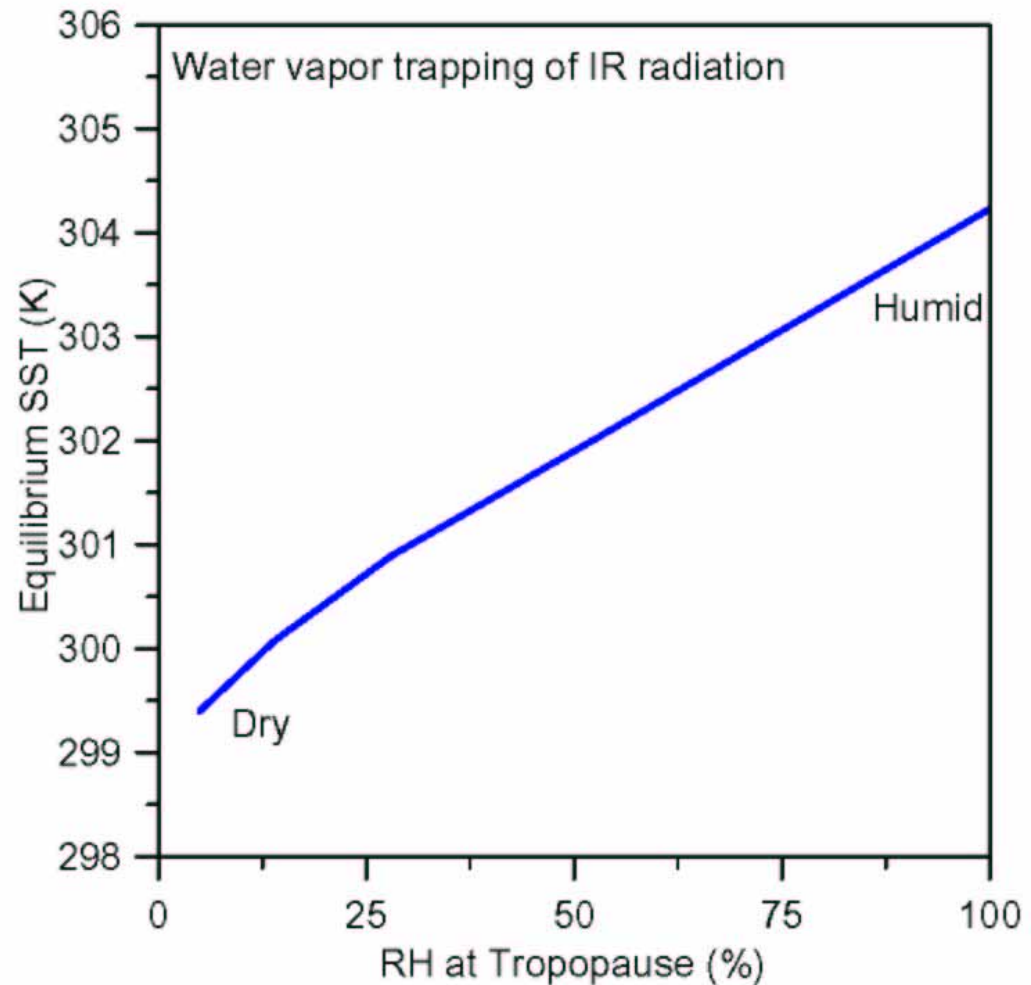


- Evaporation increases with wind *Uses 1,2*
- $\theta_E$  increases as cloud-base descends, moving towards saturation at SST

# SST equilibrium sensitive to LW

- Humid upper troposphere and equilibrium SST increases [greenhouse]

*Uses 1,2,3,4*

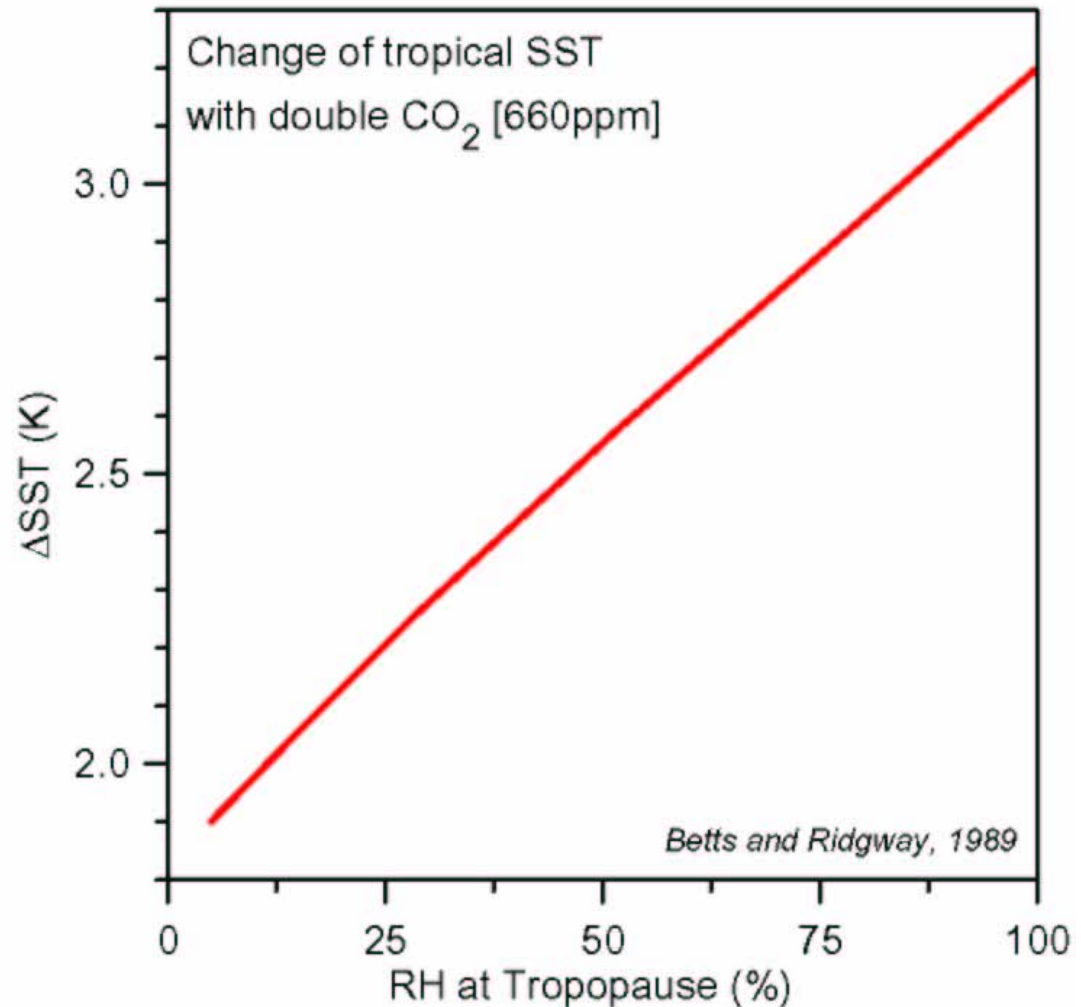




# SST sensitive to CO<sub>2</sub>

- Tropical climate sensitivity approx 2K for doubling of CO<sub>2</sub>
- Sensitivity increases if upper troposphere moist

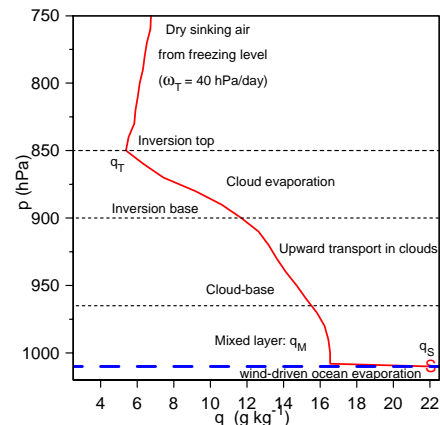
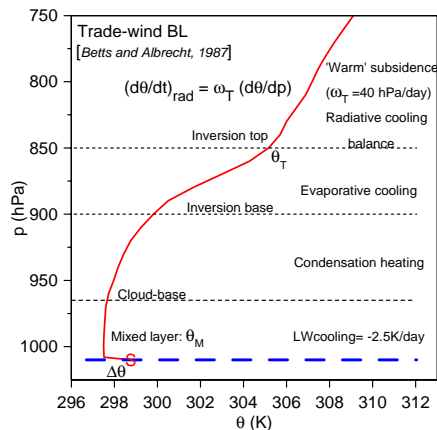
*Uses 1,2,3,4*



# How does ocean BL and land differ?

Radiative cooling  
SH, subsidence

Evaporation and  
subsidence



Stays a little cooler than ocean and sub-saturated:  
surface wind and subsidence control evaporation  
[ocean store sun's heat; diurnal cycle small]

## LAND: what are the essential differences??

Sun heats surface and drives large diurnal cycle; daytime unstable;  
cools radiatively at night; at night stable BL

Surface not saturated.. Except inside leaves.

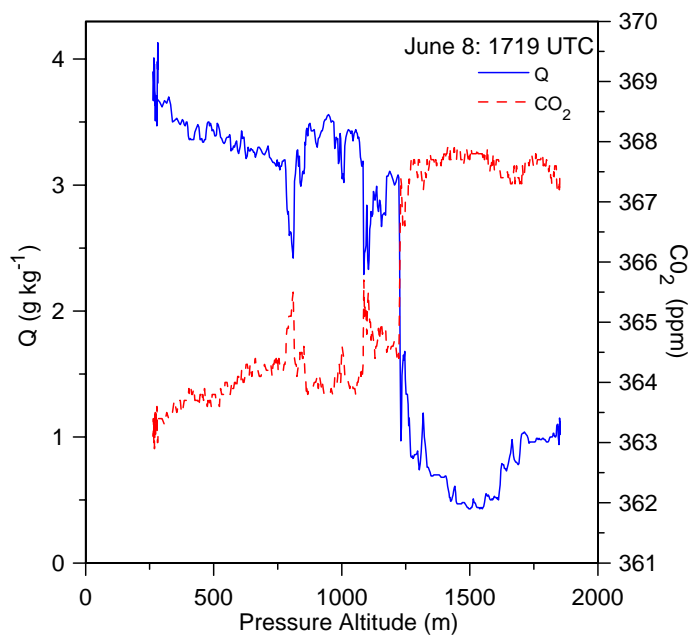
Sun drives evaporation through photosynthesis  
[coupled to CO<sub>2</sub> uptake]

Subsidence of dry air still plays key role, averaged over 24hrs.

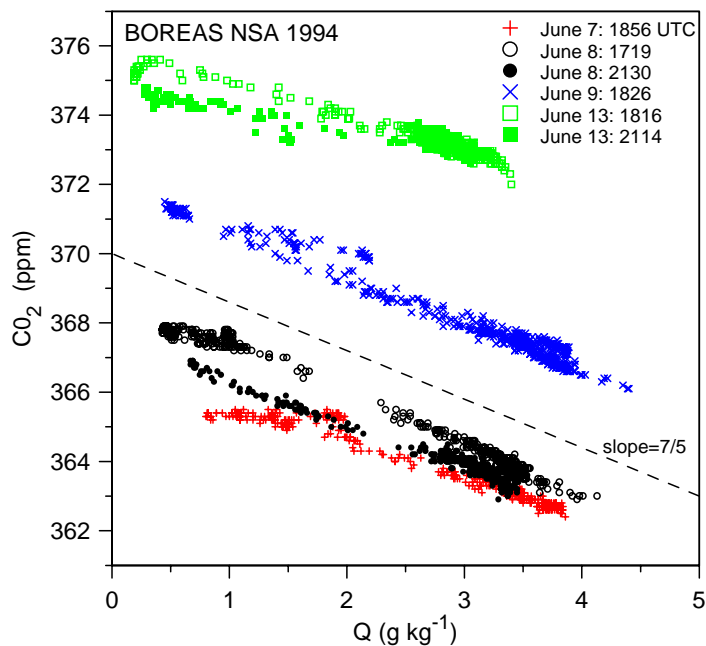
*Need to understand mean state and diurnal cycle*

# Coupling of CO<sub>2</sub> and water vapor through the BL

BOREAS Northern Study area [Thompson, Manitoba]



**Figure 1** Coupling of CO<sub>2</sub> and water vapor profiles of June 8 at 1719 UTC (LST=UTC-6h)

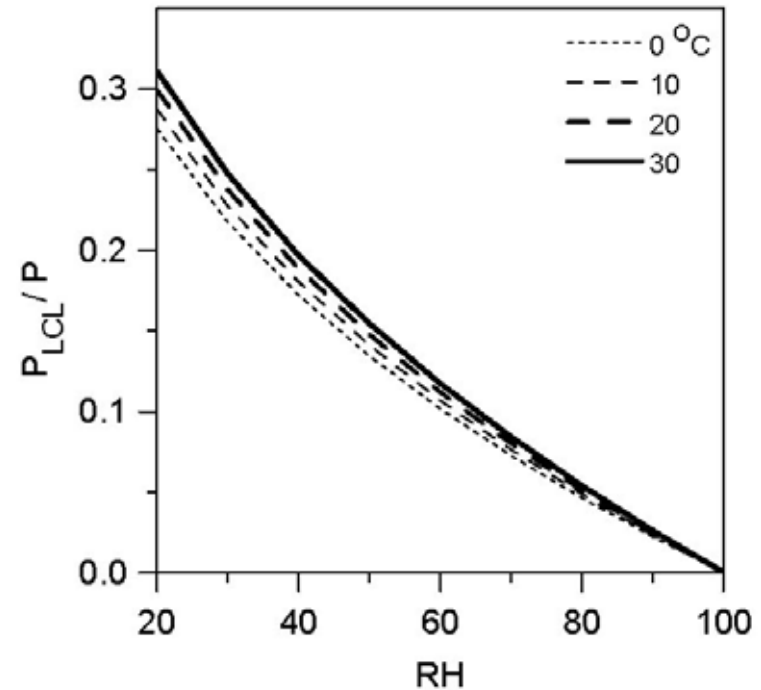
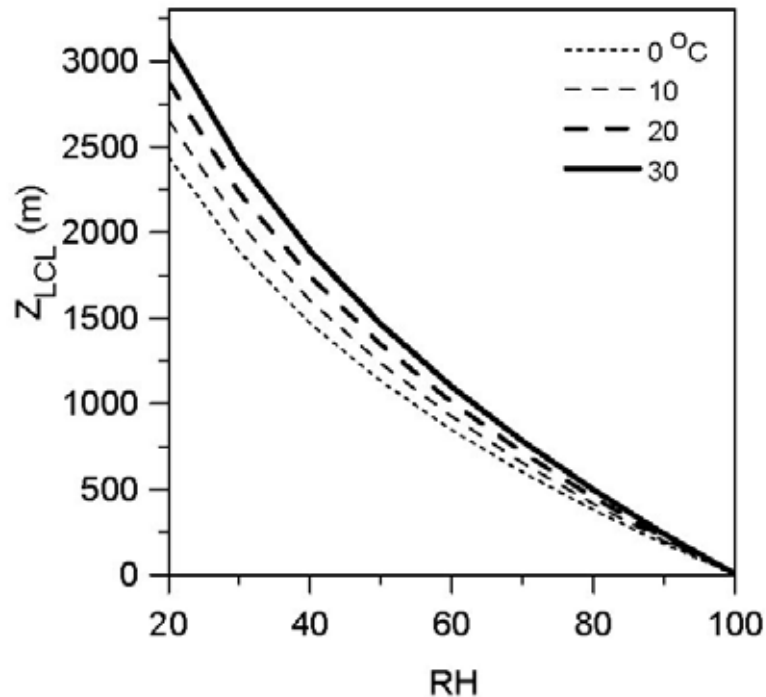


**Figure 2.** Profiles through the mixed layer on four days in June, showing tight coupling between water vapor and CO<sub>2</sub> structure. Illustrative slope of 7 ppm CO<sub>2</sub> to 5 g kg<sup>-1</sup> is shown.

# Daily mean fluxes give model 'equilibrium climate' state

- Map model climate state and links between processes using daily means
- Think of seasonal cycle as transition between daily mean states  
+ synoptic noise

SMI  $\longrightarrow$   $R_{veg}$   $\longrightarrow$  RH  $\longrightarrow$  LCL  $\longrightarrow$  LCC



- RH gives LCL [largely independent of T]
- Saturation pressure conserved in adiabatic motion
- Think of RH linked to availability of water

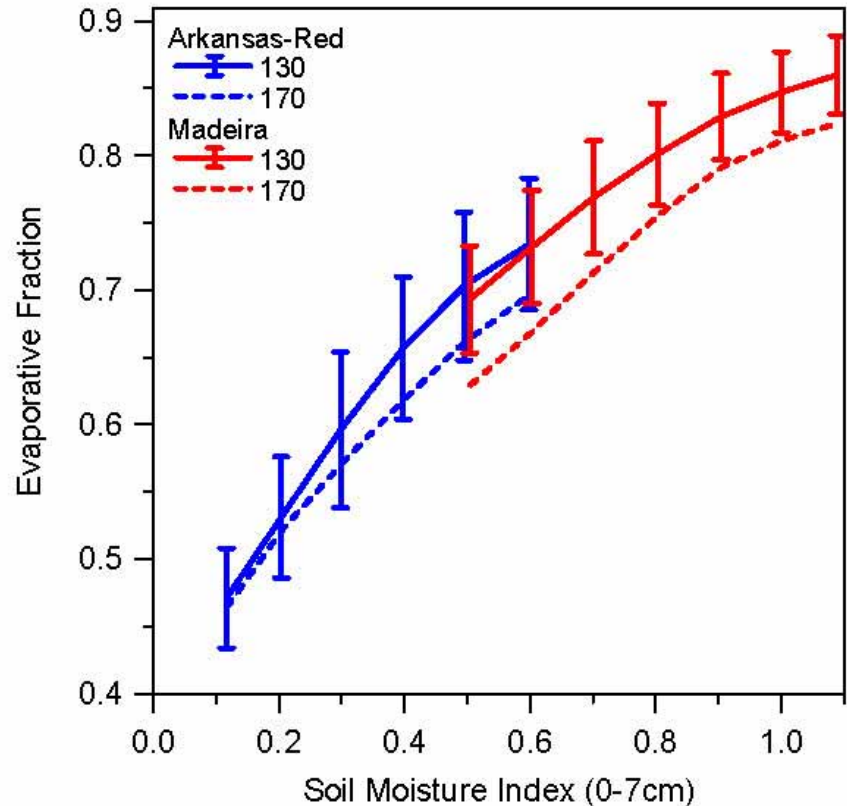
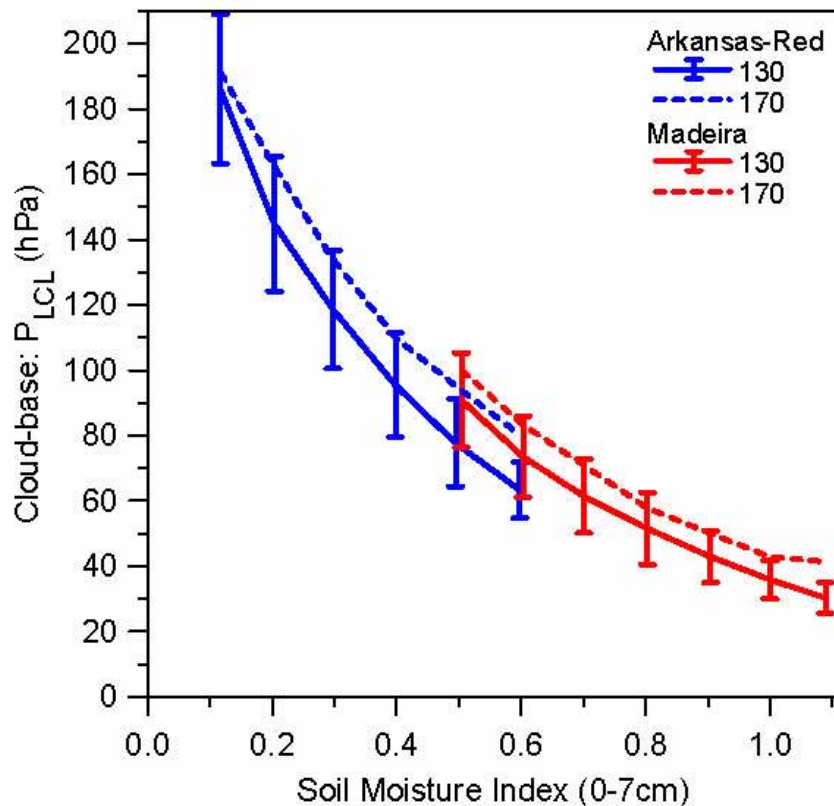
# What controls daily mean RH anyway?

- RH is balance of subsidence velocity and surface conductance
- Subsidence is radiatively driven [40 hPa/day] + atmospheric dynamics
- Surface conductance

$$G_s = G_a G_{veg} / (G_a + G_{veg}) \quad [G_{veg} = 1/R_{veg}]$$

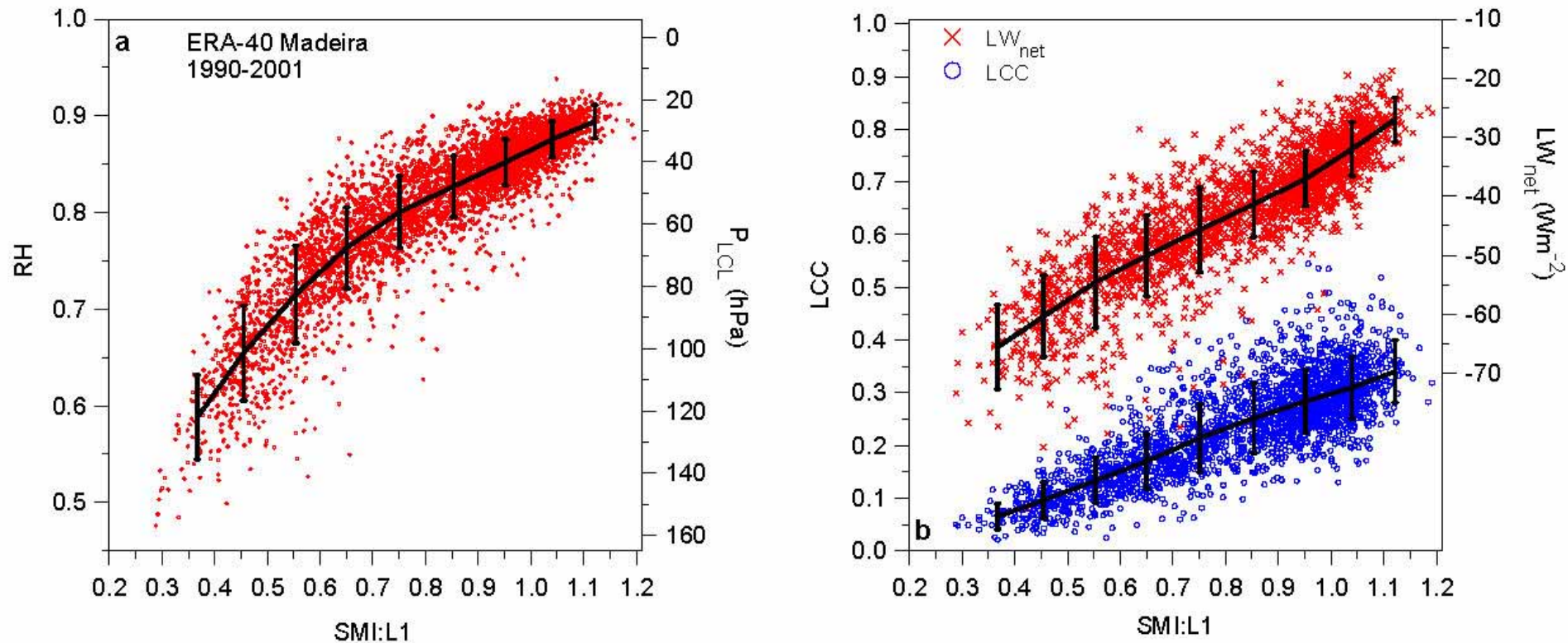
[30 hPa/day for  $G_a = 10^{-2}$ ;  $G_{veg} = 5 \cdot 10^{-3}$  m/s]

# ERA40: soil moisture $\rightarrow$ LCL and EF



- River basin daily means
- Binned by soil moisture and  $R_{net}$

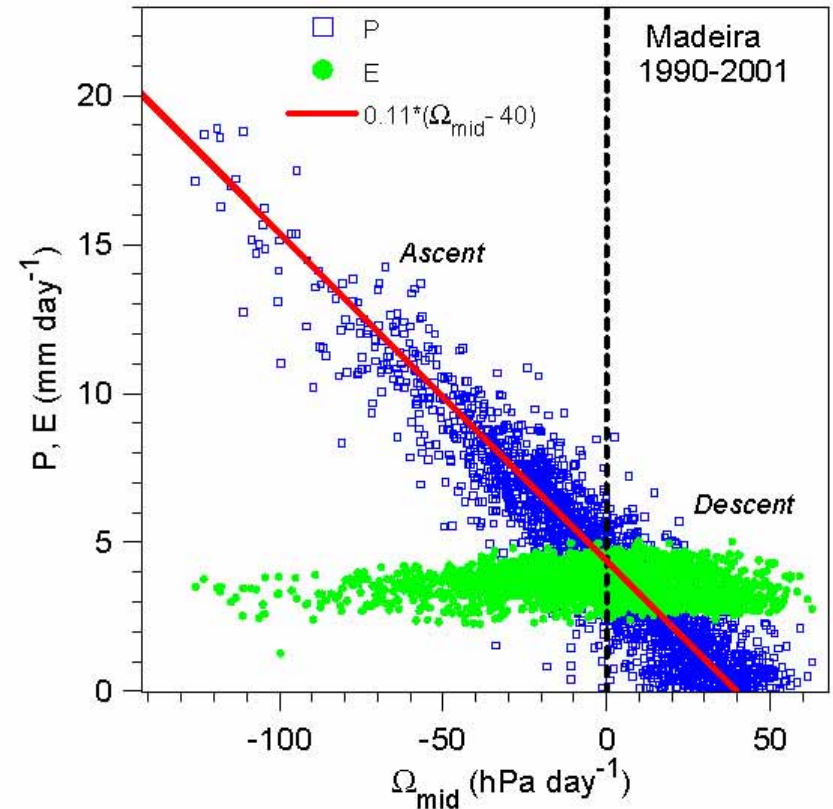
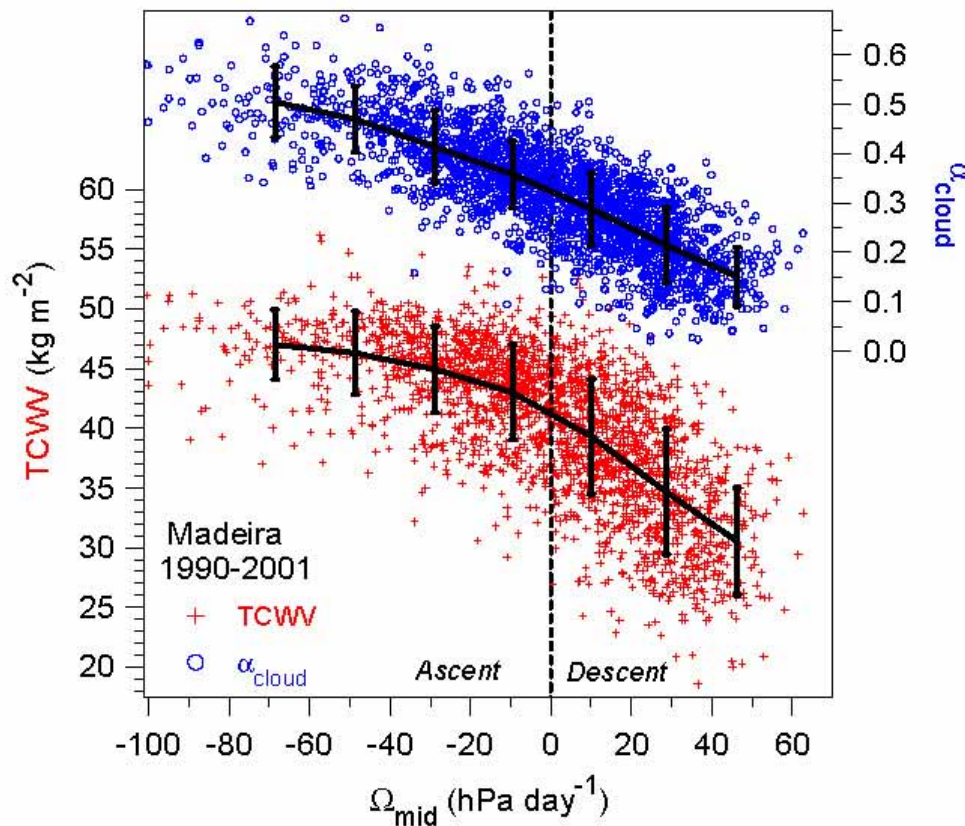
# ERA40: Surface 'control'



- Madeira river, SW Amazon
- Soil water  $\rightarrow$  LCL, LCC and  $LW_{net}$



# ERA-40 dynamic link (mid-level omega)



- $\Omega_{\text{mid}} \rightarrow$  **Cloud albedo**, TCWV and Precipitation

# Take away these ideas

*Ocean equilibrium*: balance of radiative cooling, subsidence and surface fluxes

giving a typical tradewind BL with cloud-base 50hPa above surface and a 150hPa deep shallow cumulus layer....

[Solar heating absorbed in deep ocean mixed layer]

Fluxes and BL  $\theta_E$  go up with SST and wind-speed

*Land diurnal cycle* driven by solar heating, but *equilibrium* similar to ocean, except a drier mean state because additional ‘vegetative’ resistance to evaporation at surface.

*CO<sub>2</sub> and water vapor coupled in BL over vegetation.*